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Implicit Curves and Tangent Lines with TI–Nspire CX CAS

Michel Beaudin, michel.beaudin@etsmtl.ca
Anouk Bergeron–Brlek, anouk.bergeron–brlek@etsmtl.ca
Louis–Xavier Proulx, louis–xavier.proulx@etsmtl.ca

École de technologie supérieure (ÉTS), Montréal, Québec, Canada

Outline

1. Tangent line to a curve $y = f(x)$ at $x = a$

2. Tangent line to a curve $f(x, y) = 0$ at the point (a, b)

3. Implicit 2D plotting: workarounds

4. Concluding remarks

1. Tangent line to a curve $y = f(x)$ at x=a

Example 1: Find the equation of the tangent line of the function

$f(x):=x^3-3\cdot x-1$ at $x = 2$.

▸ **Solution of Ex 1:**

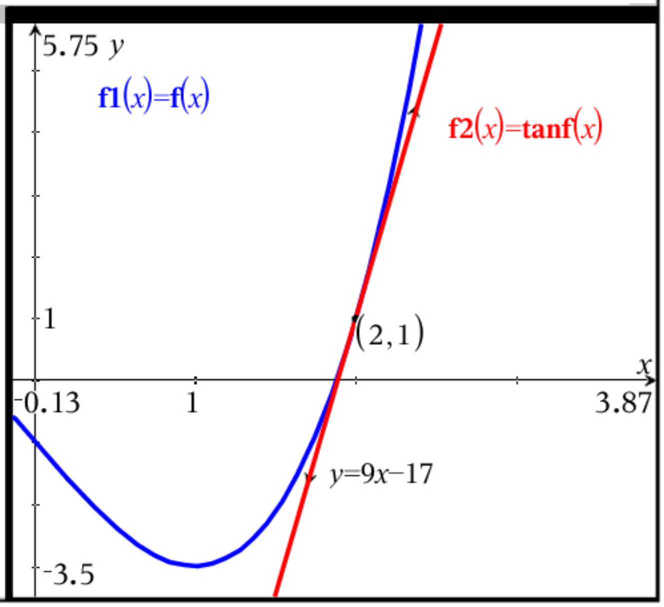
a) The tangent line to the curve $y = f(x)$ at $x = a$ is given by $y(x) = f(a) + f'(a)\cdot(x-a)$.

$\text{tanf}(x):=1+9\cdot(x-2)$ ▸ *Done*

b) Use the built−in function.

$\text{tangentLine}(\text{f}(x),x=2)$ ▸ $9\cdot x-17$

c) Use the built−in geometry package.



2. Tangent line to a curve $f(x, y) = 0$ at the point (a, b)

Example 2: Plot the circle $x^2 + y^2 - 25 = 0$ and the tangent line at the point (3, 4).

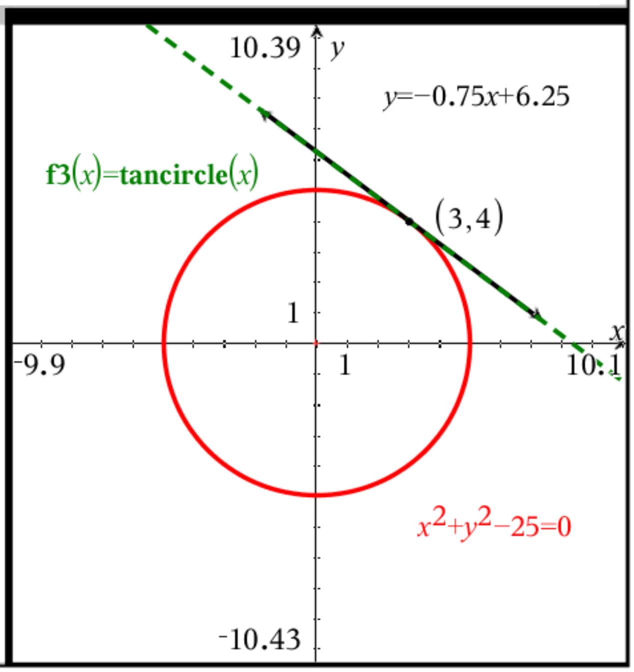
▸ **Solution of Ex 2:**

a) Implement a function using the built−in function **impdif()**

$\text{impDif}(x^2+y^2-25=0,x,y)|x=3 \text{ and } y=4$ ▸ $\frac{-3}{4}$

$\text{tancircle}(x):=4+\frac{-3}{4}\cdot(x-3)$ ▸ *Done*

b) Plot the circle as a relation and the tangent line.



2. Tangent line to a curve $f(x, y) = 0$ at the point (a, b)

Example 2: Plot the circle $x^2 + y^2 - 25 = 0$ and the tangent line at the point (3, 4).

c) Solve the relation for the variable y and plot each solution.

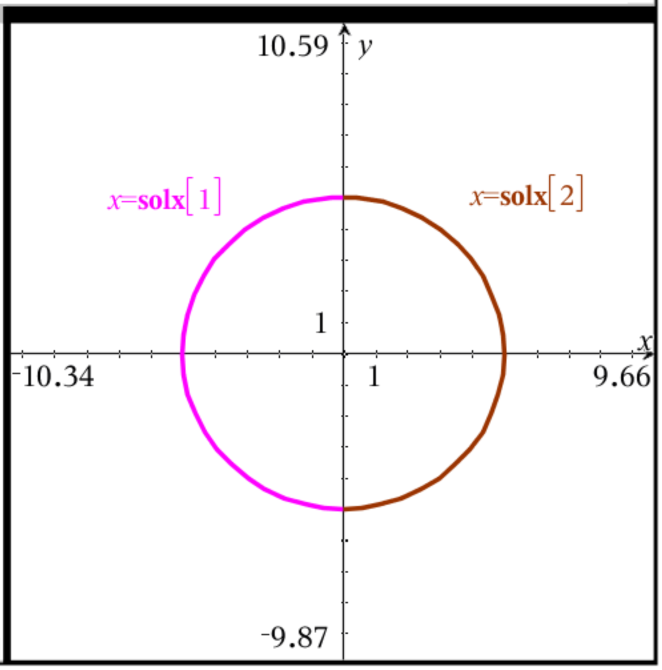
$\mathbf{soly} := \text{zeros}(x^2+y^2-25, y)$

$\blacktriangleright \left\{ \left\{ -\sqrt{25-x^2}, x^2 \le 25, \left\{ \sqrt{25-x^2}, x^2 \le 25 \right\} \right.\right.$

d) Solve the relation for the variable x and plot each solution.

$\mathbf{solx} := \text{zeros}(x^2+y^2-25, x)$

$\blacktriangleright \left\{ \left\{ -\sqrt{25-y^2}, y^2 \le 25, \left\{ \sqrt{25-y^2}, y^2 \le 25 \right\} \right.\right.$



2. Tangent line to a curve $f(x, y) = 0$ at the point (a, b)

Example 2: Plot the circle $x^2 + y^2 - 25 = 0$ and the tangent line at the point (3, 4).

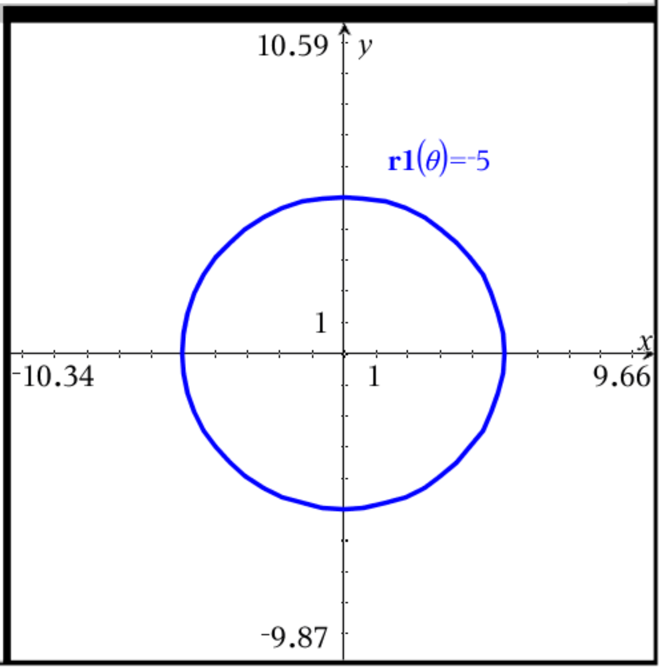
▸ **Solution of Ex 2:**

e) Use the polar plot mode.

$x^2+y^2-25=0 \mid x=r\cdot\cos(\theta) \text{ and } y=r\cdot\sin(\theta)$

$\blacktriangleright r^2-25=0$

$\text{zeros}(r^2-25, r) \blacktriangleright \{-5, 5\}$

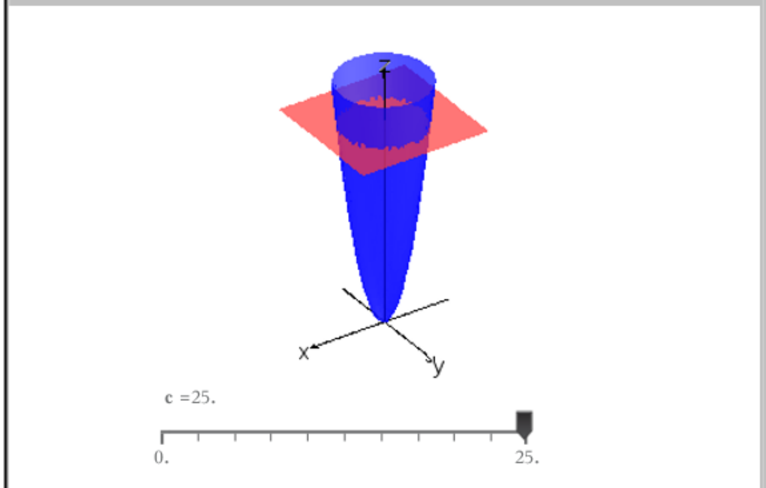


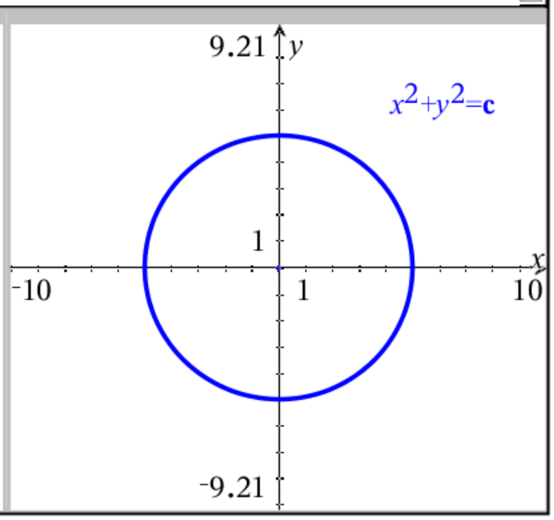
2. Tangent line to a curve $f(x, y) = 0$ at the point (a, b)

Example 2: Plot the circle $x^2 + y^2 - 25 = 0$ and the tangent line at the point (3, 4).

▸ **Solution of Ex 2:**

f) Intersect the 3D graph of $\mathbf{f}(x,y)=x^2 + y^2$ with the plane $z=25$.





3. Implicit 2D plotting: workarounds

W1) Solve for *y* and plot the solutions

or

Use zeros($f(x,y), y$) in a 2D graphics mode window.

W2) W1) and select the rectangular mode in the Document Settings.

W3) Solve for *x* and plot the solutions as relations.

W4) Use the polar plot mode.

W5) Intersect the 3D graph of $f(x,y)=0$ with the plane $z=0$.

W1) 2D Graphics mode window

Example 3: Plot the relation $\arctan(x\cdot y)-2x-y=0$ and compute the tangent line at $x=-1$.

▸ **Solution for Ex 3:**

$\text{zeros}(\tan^{-1}(x\cdot y)-2\cdot x-y,y)$ ▸ { ⬚ }⚠

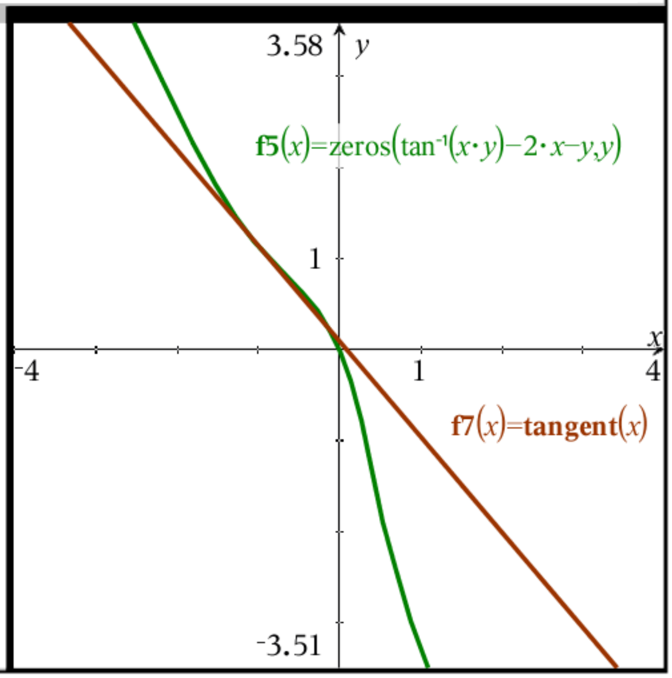
Tangent line at $x=-1$

$\text{impDif}(\tan^{-1}(x\cdot y)-2\cdot x-y=0,x,y)$

$\textbf{y\_val}:=(\text{zeros}(\tan^{-1}(x\cdot y)-2\cdot x-y,y))|x=-1$

$\textbf{slope}:=\text{impDif}(\tan^{-1}(x\cdot y)-2\cdot x-y=0,x,y)|x=-1 \text{ and } y=\textbf{y\_val}$

$\textbf{tangent}(x):=\textbf{slope}\cdot(x+1)+\textbf{y\_val}$ ▸ *Done*



W2) Rectangular mode

Example 4: Plot the curve $x^3\cdot y^2-3\cdot x^2\cdot y^3+20=0$.

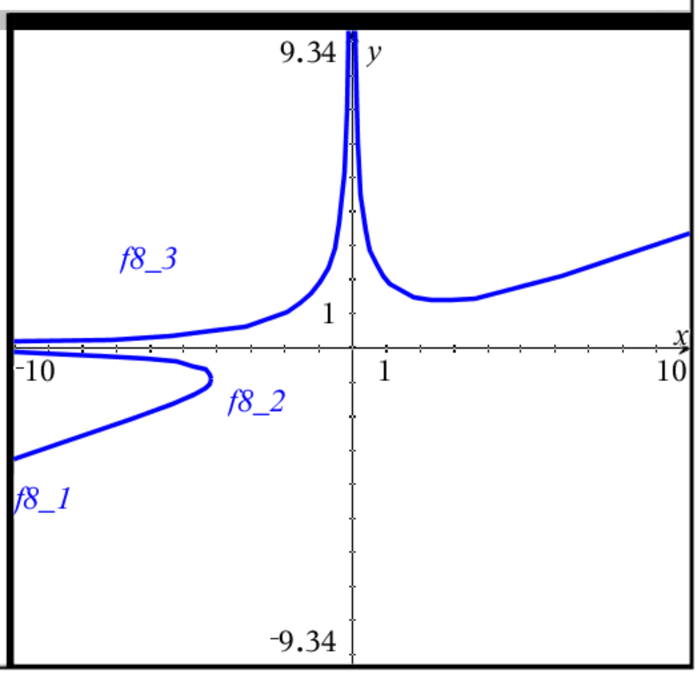
▸ **Solution for Ex 4:**

W1) Part of the curve is missing.

$\textbf{solex4}(x):=\text{zeros}(x^3\cdot y^2-3\cdot x^2\cdot y^3+20,y)$ ▸ *Done*

$\textbf{solex4}(x)[2]$

▸ $$\frac{2\cdot|x|\cdot\sin\left(\frac{\sin^{-1}\left(\frac{x^5+2430}{|x^5|}\right)}{3}+\frac{\pi}{3}\right)+x}{9}$$ ⚠



W3) Relation plot mode/inverse function

Example 5: Plot the relation $3\,x\cdot y-2\,x^{2}+4\,\sin(y)+6=0$.

$\text{zeros}(3\cdot x\cdot y-2\cdot x^{2}+4\cdot\sin(y)+6,y)$

▸ { ⬚ } ⚠

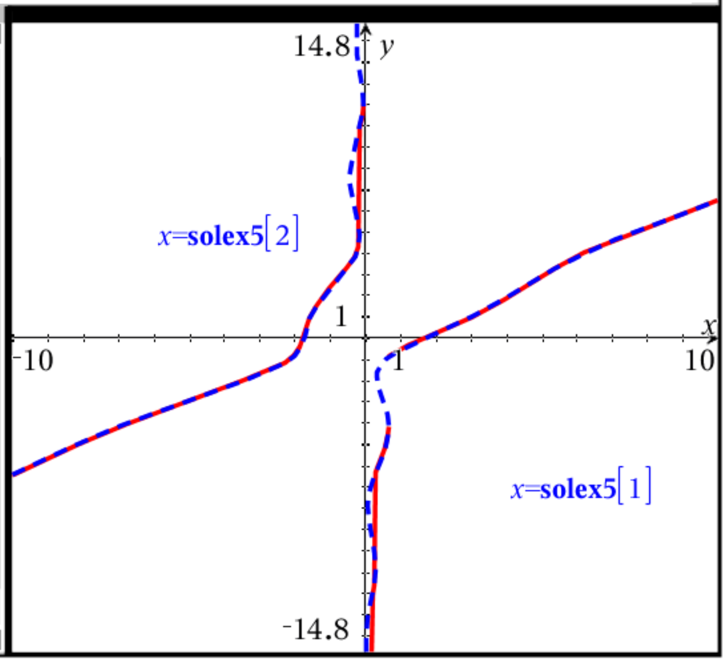
W3) The equation

$3\,x\cdot y-2\,x^{2}+4\,\sin(y)+6=0$ can be solved for x.

solex5

$:=\text{zeros}(3\cdot x\cdot y-2\cdot x^{2}+4\cdot\sin(y)+6,x)$

▸ $\left\{\frac{\sqrt{32\cdot\sin(y)+9\cdot y^{2}+48}+3\cdot y}{4},\frac{-(\sqrt{32\cdot s\ldots}}{}\right.$

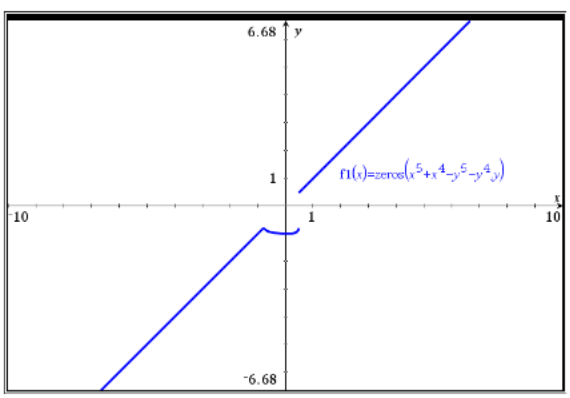


W4) Polar plot mode

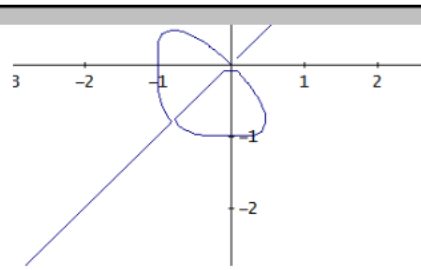
Example 6: Plot the relation $x^{5}+x^{4}=y^{5}+y^{4}$.

Nspire CX CAS

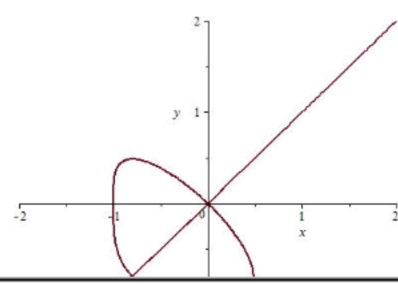
W2) Part of the curve is missing.



W3) $\text{zeros}(x^{5}+x^{4}-y^{5}-y^{4},x)$ ▸ { ⬚ } ⚠



Maple



W4) Polar plot mode

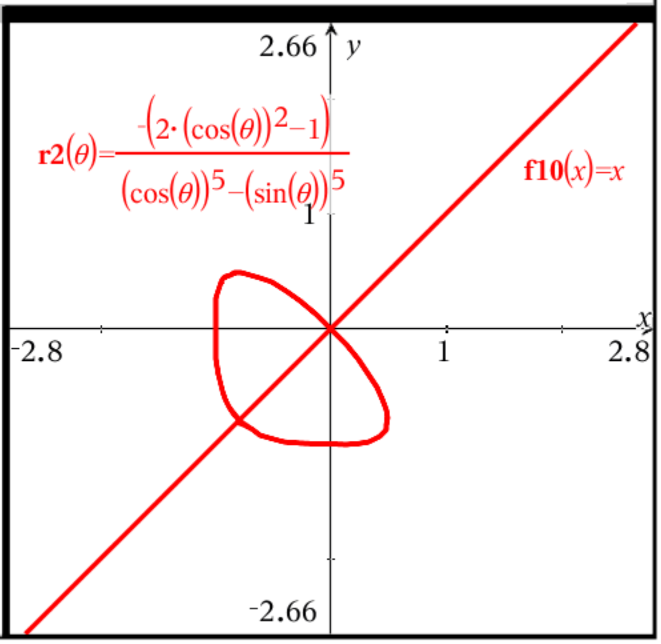
Example 6: Plot the relation $x^5+x^4=y^5+y^4$.

▸ **Solutions for Ex 5:**

Polar coordinates

$\mathbf{ex6}:=x^5+x^4=y^5+y^4|x=r\cdot\cos(\theta)$ and $y=r\cdot\sin(\theta)$

$\text{solve}(\mathbf{ex6},r) \blacktriangleright r=\frac{-(2\cdot(\cos(\theta))^2-1)}{(\cos(\theta))^5-(\sin(\theta))^5}$ or $r=0$



W5) 3D plot function mode

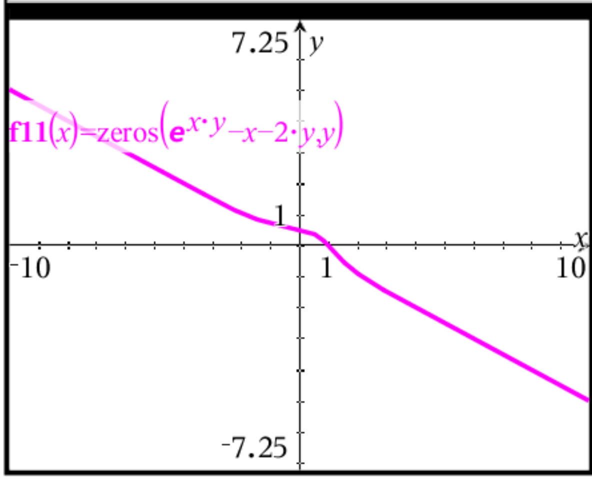
Example 7: Plot the curve $\mathbf{e}^{x\cdot y}-x-2\,y=0$. Find all points (x,y) where $x=1$.

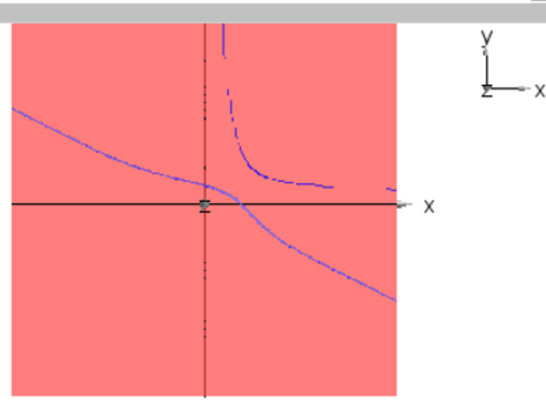
▸ **Solution of Ex 7:**

W2) Part of the curve is missing. $\mathbf{f11}(1)$

W3) Fails. $\text{zeros}(\mathbf{e}^{x\cdot y}-x-2\cdot y,x) \blacktriangleright \{\square\}$ ⚠

W4) Polar plot fails.





4. Concluding remarks

- Built−in impdif() function for computing tangent lines.

- Encountered issues in implicit plotting:
  - Cannot plot the tangent line with the geometry menu.
  - Part of the curve missing.
  - Wrong representation of the curve.
  - Limitations with the handheld device CPU.

- Workarounds:
  - Five workarounds.
  - Suggestion: in any case, start with a 3D representation.

4. Concluding remarks

- We advice TI developpers to implement a dedicated 2D implicit plotter on TI−Nspire CX CAS (fast CPU). This is a **must** in calculus as well as in differential equations.

- TI−Nspire CX CAS 2D plotting algorithm is not the same as the TI Voyage 200. V200 was able to plot **any** implicit curve $\mathbf{f}(x, y) = 0$ using $\text{zeros}(\mathbf{f}(x,y), y)$ in a 2D plot function mode window. Major drawback: slow CPU.

- Since the implicit plotter of V200 was robust, why not importing it to TI−Nspire CX CAS?