

L'HOSPITAL'S WEIGHT PROBLEM

Crossing ~~a new~~ the Border



... into Mexico

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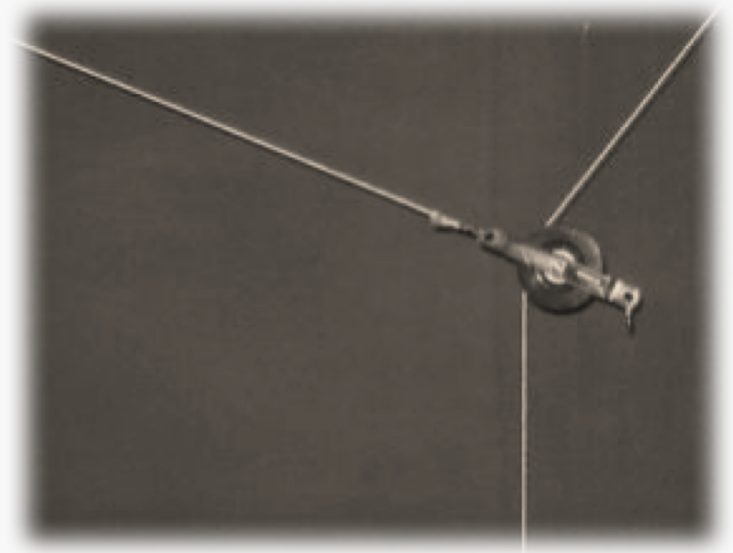
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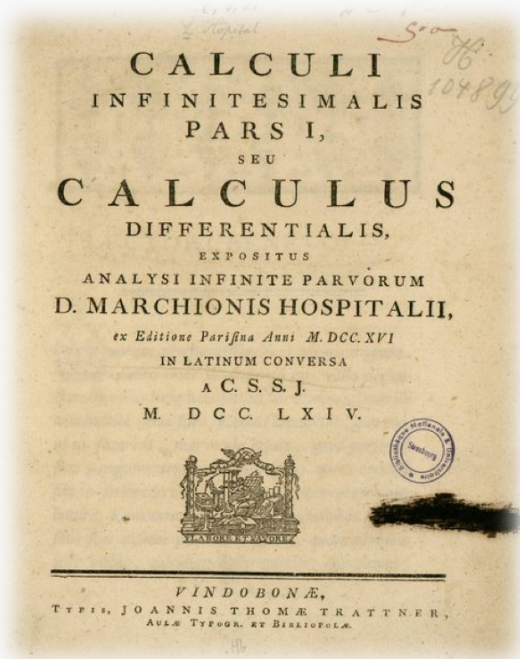
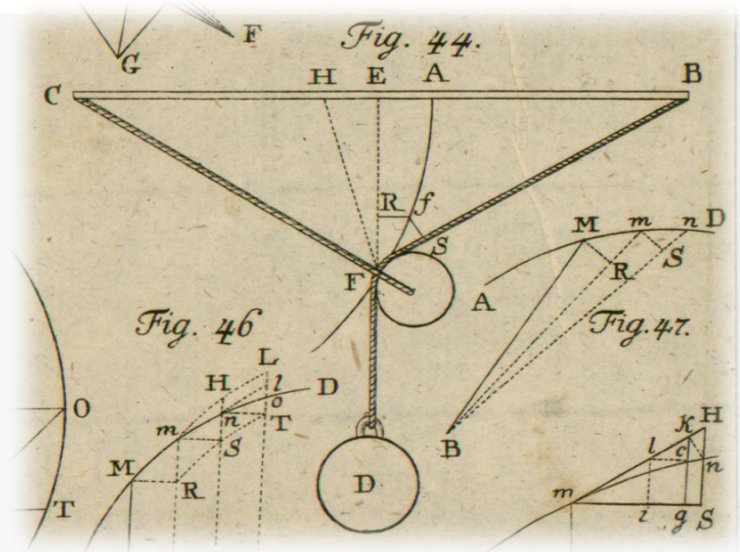


Outline



- The original weight problem
- The activity
- Its adaptation and use in Mexico
- Observations
- Conclusions

From Bernoulli to L'Hospital



To introduce differential calculus...

The activity and its variations



- Initially envisioned for high school
 - ✦ Hands-on activity
 - ✦ Synthesis of knowledge
- Designed for the transition course at ÉTS (engineering school)
 - ✦ By physics lecturer, math lecturer, and math educator
 - ✦ Review of secondary mathematics (trig), leading to calculus
 - ✦ Introduction to modelling
- Reused at the college level
 - ✦ In both pre-university and technical programs
- **Shortened and used in high school (Montreal & Mexico)**



ETS Bordereaux électroniques - Liste des membres par équipe
Cours groupe : MAT144-01 Trimestre : automne 2011 Activité : Projet1

Numéro de l'équipe	Nom, prénom	Courriel
01	André, Étienne	andree.andre@etsu.ca
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- Teams of 2 or 3
- Each team had
 - its own apparatus
 - measuring instruments
 - symbolic calculators (TI Nspire – CAS)

Overview of the (whole) Activity



1. Exploration



2. Constructing Equations

[1e] $\cos \theta = \frac{D - a \cos \alpha}{b}$

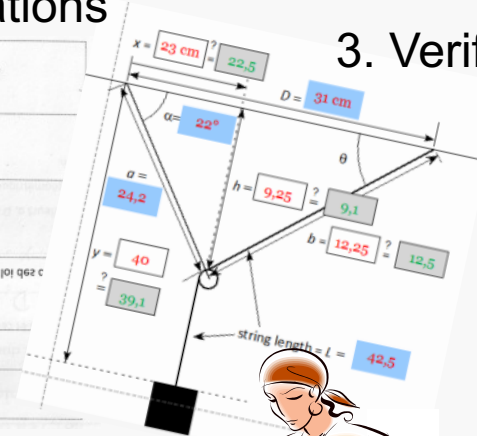
[1r] $\sin \theta = \frac{p}{a \sin \alpha}$

[1v] $\lambda = \frac{a \sin \theta}{\sin \alpha} + 1 = \frac{a \sin \theta + a \sin \alpha}{a \sin \alpha}$

[1s] $p = a \sin \theta = D \sin \alpha$

[1t] $x = a \cos \theta$

3. Verification



$0 = a \cdot \cos(\alpha) - a \cdot D \cdot \sin(\alpha)$

$\sqrt{-2 \cdot a \cdot D \cdot \cos(\alpha) + a^2 + D^2}$

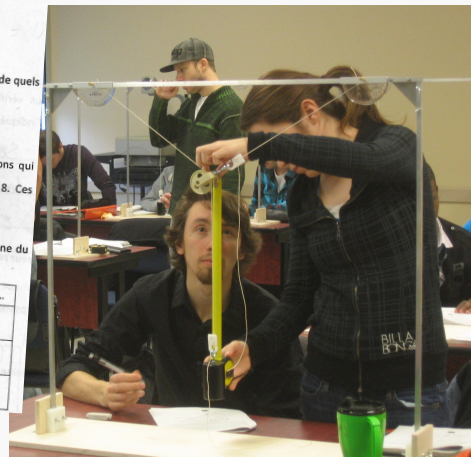
La solution de cette équation, lorsque résolue pour α (ne résolvez pas l'équation), dépendra de quels paramètres ?

α dépend des paramètres: a, D (3.1)

Réécrivez dans la deuxième colonne du tableau ci-dessous les formes générales des fonctions qui décrivent la position de la poulie (x, h) et du poids (x, y) . Voyez le tableau 1 de la page 8. Ces expressions ne devraient contenir que α, a, D et L .

Sachant que α ne dépend que des paramètres indiqués en (3.1), complétez la troisième colonne du tableau ci-dessus en examinant bien les formules pour x, h et y .

		dépend des paramètres...
poulie	$x = a \cos(\alpha)$	a, D
	$h = a \cdot \sin(\alpha)$	a, D
poids	$y = a \sin(\alpha) + L - \sqrt{-2 \cdot a \cdot D \cdot \cos(\alpha) + a^2 + D^2}$	a, D, L



4. Prediction

6. Revisiting intuitions

5. Validation

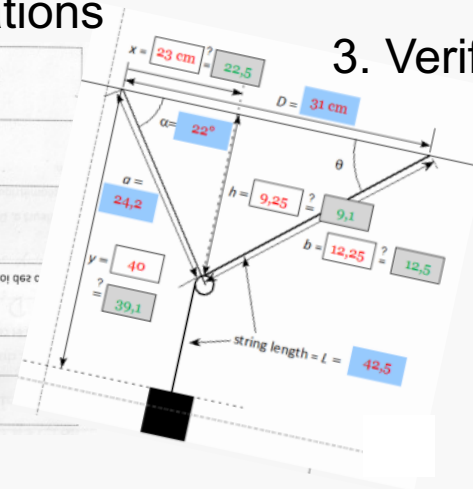
Variation for Mexico



2. Constructing Equations

$$\begin{aligned} [1e] \cos \theta &= \frac{13}{20} = \frac{13}{20} \\ [1f] \sin \theta &= \frac{p}{20} \\ [1g] \lambda &= \frac{13}{20} + 1 - \frac{13}{20} = 1 \\ [1h] p &= \sqrt{20^2 - 13^2} = 15 \\ [1i] \mu &= \frac{p}{20} = \frac{15}{20} = \frac{3}{4} \\ [1j] x &= 20 \cos \theta \end{aligned}$$

3. Verification



CCH, UNAM & IPN

- bachillerato: 15 to 17 year-olds
- groups of about 25 students,
- in 2nd or 4th semesters
- 2 or 3 teachers participated in the activity each time

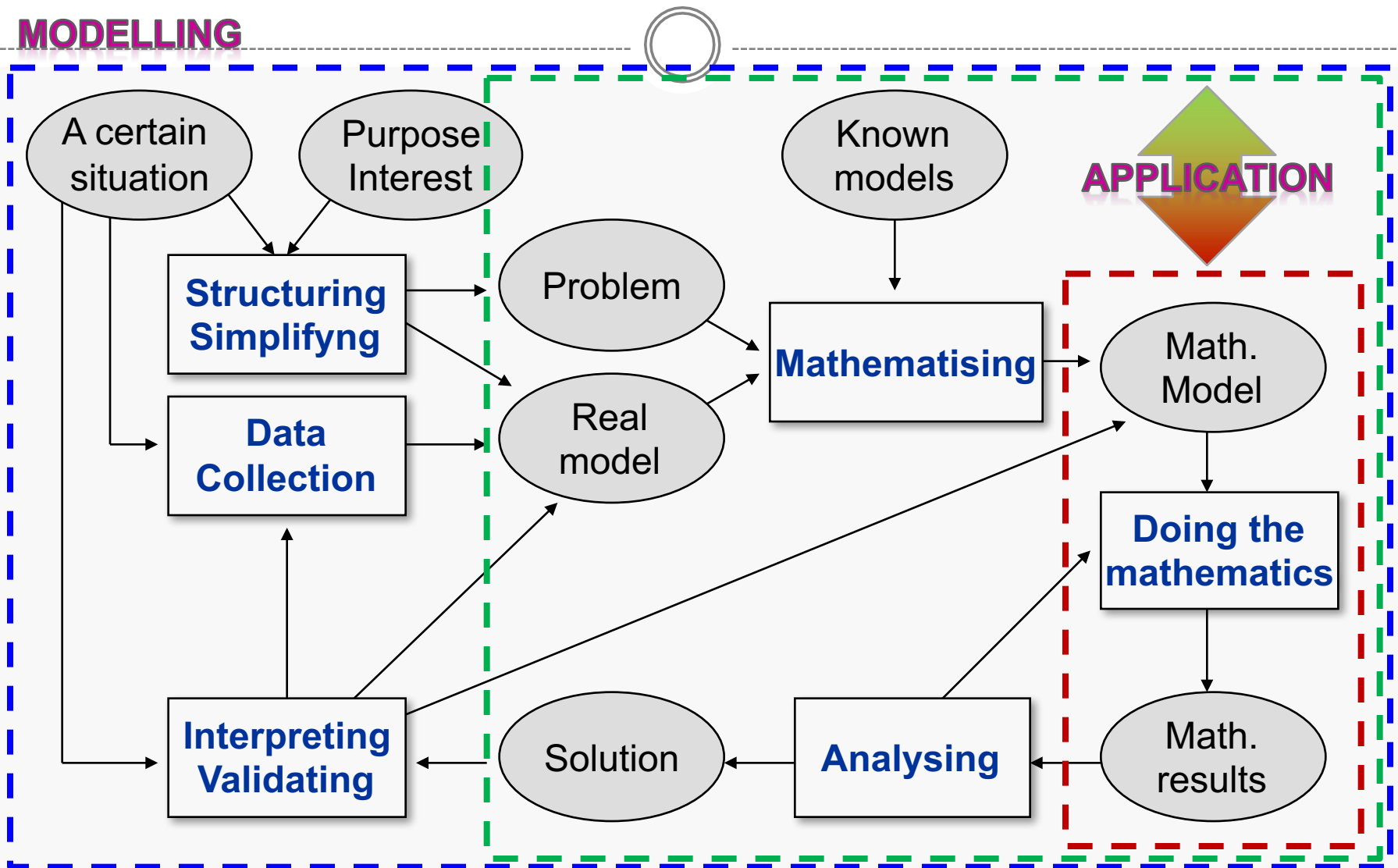


**CCH – Sur
IPN**

**Teams of 3 to 5 students
2 apparatus with measuring tapes
Each team had its own calculator (phone, ...)**

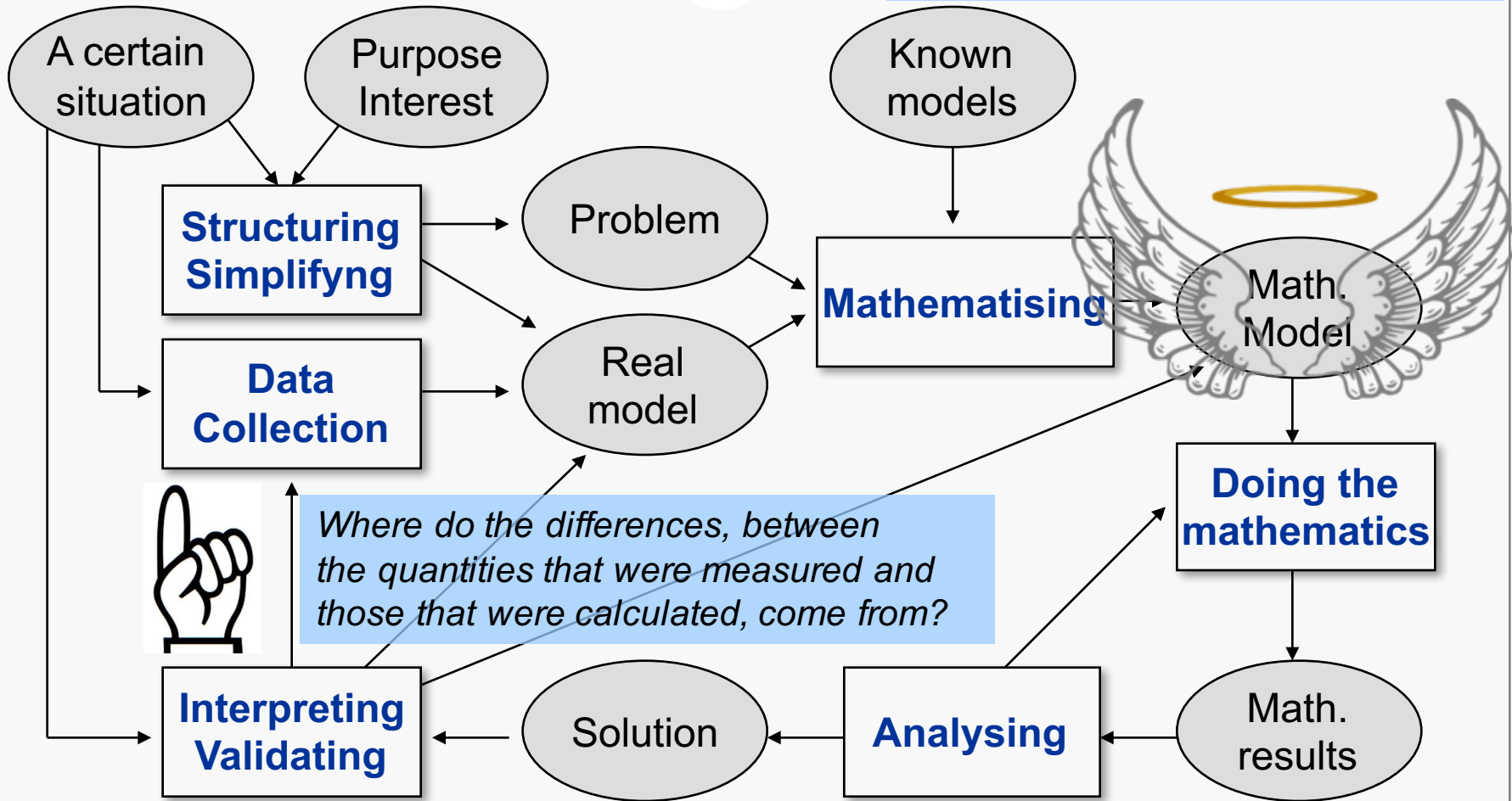
The Modelling Cycle (adapted from Blum & al., 2002)

MODELLING

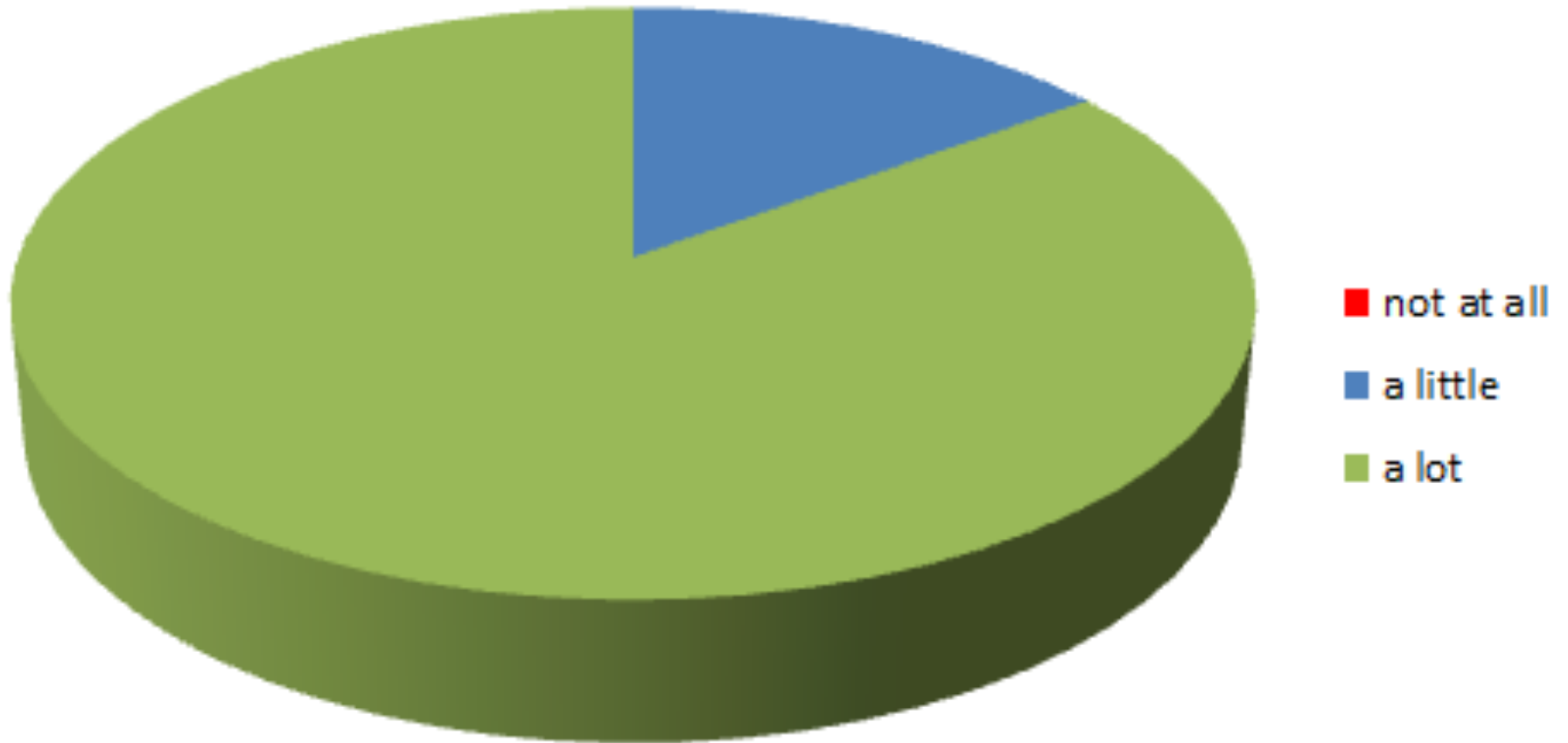


The Modelling Cycle (adapted from Blum & al., 2002)

“las medidas son una predicción y lo calculado es exacto”



The Student's Perspective



The Student's Perspective



What I liked

- **Theory** put in **practice**. **Application** in real life.
- **Manipulation**. Concrete, visual. **Interactive**.
- History.
- **Different approach** than **physics**.
- **Learning Guide**. **Progressive** approach.
- Possibility to **review** and **understand**, to **validate** and **self-correct**.
- **Team dynamics**.
- Visiting teacher. **More than one teacher** during the activity.

What I didn't like

- **Measuring**.
- **Time** constraints.
- **Team dynamics**.
My team didn't cooperate. My team didn't communicate enough.
- My profs at CCH do **not usually** have us do this type of activity.
- Formulas were difficult to find.

The Student's Perspective



What I learned

- **Relationships between variables**, between side lengths and angles.
- That we can **validate** our work **mathematically**.
- Trigonometric functions. Pythagoras' theorem. The cosine property.
- To **find formulas** and to **measure**.
- To **substitute** in order to find values.
- To **think**
(due to the difficulty of finding formulas.)
- **Reviewed** themes from math and physics.

What remains unclear

- Properties of sine and cosine functions.
- **Why** there are **differences** between what is measured and what is calculated.

Observations



- Acknowledge that the model is a simplified (imperfect) representation of a real situation; to discuss assumptions, limitations,...
- Value other disciplines to revisit assumptions.

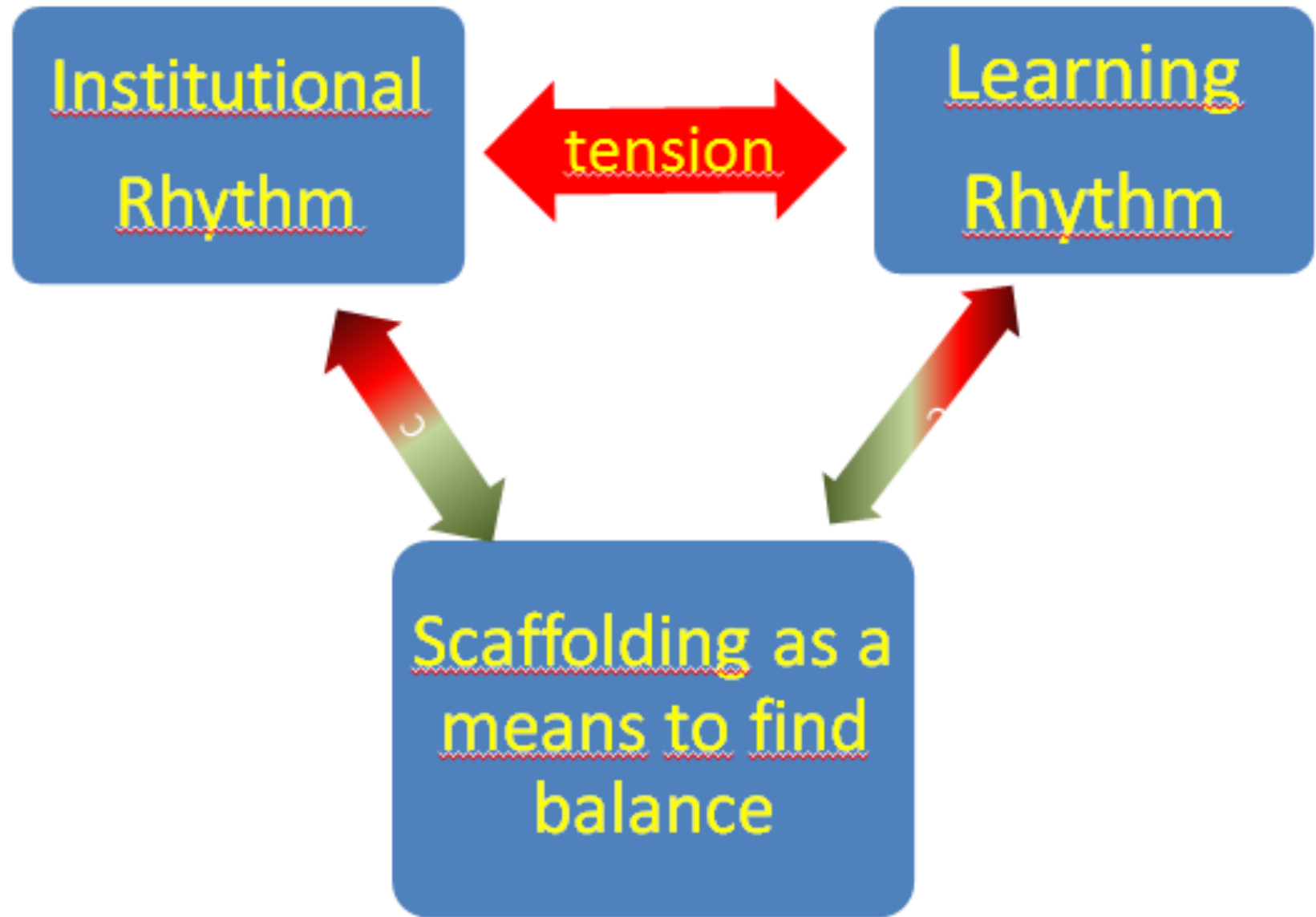
“When we deduce or find a formula and apply it, we can see if the formula is correct or not.”

Institutional
Rhythm

tension

Learning
Rhythm

Scaffolding as a
means to find
balance



Observations



Scaffolding

- Physical – structure of the activity – is key
 - ✦ to **empowering** students
 - ✦ to **avoid** their **straying** too far away from course content
- Natural – team work (Vigosky)
 - ✦ **empowers** students
 - ✦ **empowers** profs to develop and try activities
- Technology as scaffolding



Conclusions



- Modelling would benefit from Institutionalisation
- Institutionalisation would benefit from modelling

FIN



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