The influence of TIME on my teaching and research

Stephan V Joubert

Tshwane UOT

June 29, 2016

Pronunciation of names

- Many of my students in South Africa have trouble with the pronunciation of my surname. They have no trouble with "Steve", or "Stephan", but "Joubert" is another story. Variations I have heard
- I too have trouble with African names such as
- We all have a good laugh at some of my attempts until I point out that one of our politicians, first name "Tokyo", probably has the strangest surname for a white Afrikaner-Scot like me to get around:
- After I point out the literal "English" pronunciation, we all move forward.

Pronunciation of names

- Many of my students in South Africa have trouble with the pronunciation of my surname. They have no trouble with "Steve", or "Stephan", but "Joubert" is another story. Variations I have heard
- Jobt
- I too have trouble with African names such as
- We all have a good laugh at some of my attempts until I point out that one of our politicians, first name "Tokyo", probably has the strangest surname for a white Afrikaner-Scot like me to get around:
- After I point out the literal "English" pronunciation, we all move forward.

Pronunciation of names

- Many of my students in South Africa have trouble with the pronunciation of my surname. They have no trouble with "Steve", or "Stephan", but "Joubert" is another story. Variations I have heard
- Jobt
- Jauber
- I too have trouble with African names such as
- We all have a good laugh at some of my attempts until I point out that one of our politicians, first name "Tokyo", probably has the strangest surname for a white Afrikaner-Scot like me to get around:
- After I point out the literal "English" pronunciation, we all move forward.

Pronunciation of names

- Many of my students in South Africa have trouble with the pronunciation of my surname. They have no trouble with "Steve", or "Stephan", but "Joubert" is another story. Variations I have heard

- Jobt
- Jauber
- Yo bert

- I too have trouble with African names such as

- We all have a good laugh at some of my attempts until I point out that one of our politicians, first name "Tokyo", probably has the strangest surname for a white Afrikaner-Scot like me to get around:

- After I point out the literal "English" pronunciation, we all move forward.

Pronunciation of names

- Many of my students in South Africa have trouble with the pronunciation of my surname. They have no trouble with "Steve", or "Stephan", but "Joubert" is another story. Variations I have heard

- Jobt
- Jauber
- Yo bert

- I too have trouble with African names such as

- We all have a good laugh at some of my attempts until I point out that one of our politicians, first name "Tokyo", probably has the strangest surname for a white Afrikaner-Scot like me to get around:

- After I point out the literal "English" pronunciation, we all move forward.

Pronunciation of names

- Many of my students in South Africa have trouble with the pronunciation of my surname. They have no trouble with "Steve", or "Stephan", but "Joubert" is another story. Variations I have heard

- Jobt
- Jauber
- Yo bert

- I too have trouble with African names such as

- Mokoena

- We all have a good laugh at some of my attempts until I point out that one of our politicians, first name "Tokyo", probably has the strangest surname for a white Afrikaner-Scot like me to get around:

- After I point out the literal "English" pronunciation, we all move forward.

Pronunciation of names

- Many of my students in South Africa have trouble with the pronunciation of my surname. They have no trouble with "Steve", or "Stephan", but "Joubert" is another story. Variations I have heard

- Jobt
- Jauber
- Yo bert

- I too have trouble with African names such as

- Mokoena
- Moshokoa

- We all have a good laugh at some of my attempts until I point out that one of our politicians, first name "Tokyo", probably has the strangest surname for a white Afrikaner-Scot like me to get around:

- After I point out the literal "English" pronunciation, we all move forward.

Pronunciation of names

- Many of my students in South Africa have trouble with the pronunciation of my surname. They have no trouble with "Steve", or "Stephan", but "Joubert" is another story. Variations I have heard

- Jobt
- Jauber
- Yo bert

- I too have trouble with African names such as

- Mokoena
- Moshokoa

- We all have a good laugh at some of my attempts until I point out that one of our politicians, first name "Tokyo", probably has the strangest surname for a white Afrikaner-Scot like me to get around:

- After I point out the literal "English" pronunciation, we all move forward.

Pronunciation of names

- Many of my students in South Africa have trouble with the pronunciation of my surname. They have no trouble with "Steve", or "Stephan", but "Joubert" is another story. Variations I have heard

- Jobt
- Jauber
- Yo bert

- I too have trouble with African names such as

- Mokoena
- Moshokoa

- We all have a good laugh at some of my attempts until I point out that one of our politicians, first name "Tokyo", probably has the strangest surname for a white Afrikaner-Scot like me to get around:

- Sexwhale

- After I point out the literal "English" pronunciation, we all move forward.

Pronunciation of names

- Many of my students in South Africa have trouble with the pronunciation of my surname. They have no trouble with "Steve", or "Stephan", but "Joubert" is another story. Variations I have heard

- Jobt
- Jauber
- Yo bert

- I too have trouble with African names such as

- Mokoena
- Moshokoa

- We all have a good laugh at some of my attempts until I point out that one of our politicians, first name "Tokyo", probably has the strangest surname for a white Afrikaner-Scot like me to get around:

- Sexwhale

- After I point out the literal "English" pronunciation, we all move forward.

Structure of this talk

- People who have influenced my teaching and research
- My career before TIME
- My career after TIME
- Throughout I will discuss some low cost real-life classroom exercises for under- and post graduates that have pointed me in the direction of some high-level technical publications.
- Throughout I will discuss
  - Human resource development
  - Technical and education publications resulting from these presentations

Structure of this talk

- People who have influenced my teaching and research
- My career before TIME
- My career after TIME
- Throughout I will discuss some low cost real-life classroom exercises for under- and post graduates that have pointed me in the direction of some high-level technical publications.
- Throughout I will discuss
  - Human resource development
  - Technical and education publications resulting from these presentations

Structure of this talk

- People who have influenced my teaching and research
- My career before TIME
- My career after TIME
- Throughout I will discuss some low cost real-life classroom exercises for under- and post graduates that have pointed me in the direction of some high-level technical publications.
- Throughout I will discuss
  - Human resource development
  - Technical and education publications resulting from these presentations

Structure of this talk

- People who have influenced my teaching and research
- My career before TIME
- My career after TIME
- Throughout I will discuss some low cost real-life classroom exercises for under- and post graduates that have pointed me in the direction of some high-level technical publications.
- Throughout I will discuss
  - Human resource development
  - Technical and education publications resulting from these presentations

Structure of this talk

- People who have influenced my teaching and research
- My career before TIME
- My career after TIME
- Throughout I will discuss some low cost real-life classroom exercises for under- and post graduates that have pointed me in the direction of some high-level technical publications.
- Throughout I will discuss
  - Human resource development
  - Technical and education publications resulting from these presentations

Structure of this talk

- People who have influenced my teaching and research
- My career before TIME
- My career after TIME
- Throughout I will discuss some low cost real-life classroom exercises for under- and post graduates that have pointed me in the direction of some high-level technical publications.
- Throughout I will discuss
  - Human resource development
  - Technical and education publications resulting from these presentations

Structure of this talk

- People who have influenced my teaching and research
- My career before TIME
- My career after TIME
- Throughout I will discuss some low cost real-life classroom exercises for under- and post graduates that have pointed me in the direction of some high-level technical publications.
- Throughout I will discuss
  - Human resource development
  - Technical and education publications resulting from these presentations

People who have influenced my teaching career

- **Temple H. Fay** from the USM started me on my education career in applied mathematics (and continues to writes papers with me).

People who have influenced my teaching career

- **Temple H. Fay** from the USM started me on my education career in applied mathematics (and continues to writes papers with me).
- The late **Keith A. Hardie** the pure mathematician from the UCT who had amazing applied mathematics ideas (and published some with me).

People who have influenced my teaching career

- **Temple H. Fay** from the USM started me on my education career in applied mathematics (and continues to writes papers with me).
- The late **Keith A. Hardie** the pure mathematician from the UCT who had amazing applied mathematics ideas (and published some with me).
- **Igor Fedotov** who started working with me at TUT in 2009 (and started the weekly colloquiums at TUT).

People who have influenced my teaching career

- **Temple H. Fay** from the USM started me on my education career in applied mathematics (and continues to writes papers with me).
- The late **Keith A. Hardie** the pure mathematician from the UCT who had amazing applied mathematics ideas (and published some with me).
- **Igor Fedotov** who started working with me at TUT in 2009 (and started the weekly colloquiums at TUT).
- **Michael Y. Shatalov** formerly from the CSIR and now at TUT who introduced me to modern gyroscopes (my coauthor on a book in production about gyroscopes).

People who have influenced my teaching career

- **Temple H. Fay** from the USM started me on my education career in applied mathematics (and continues to writes papers with me).
- The late **Keith A. Hardie** the pure mathematician from the UCT who had amazing applied mathematics ideas (and published some with me).
- **Igor Fedotov** who started working with me at TUT in 2009 (and started the weekly colloquiums at TUT).
- **Michael Y. Shatalov** formerly from the CSIR and now at TUT who introduced me to modern gyroscopes (my coauthor on a book in production about gyroscopes).
- **Joseph Bohm** who always had something new and exciting together with some real mathematics to talk about (and plays a great game of tennis).

People who have influenced my teaching career

- **Temple H. Fay** from the USM started me on my education career in applied mathematics (and continues to writes papers with me).
- The late **Keith A. Hardie** the pure mathematician from the UCT who had amazing applied mathematics ideas (and published some with me).
- **Igor Fedotov** who started working with me at TUT in 2009 (and started the weekly colloquiums at TUT).
- **Michael Y. Shatalov** formerly from the CSIR and now at TUT who introduced me to modern gyroscopes (my coauthor on a book in production about gyroscopes).
- **Joseph Bohm** who always had something new and exciting together with some real mathematics to talk about (and plays a great game of tennis).
- **Carl Leinbach** who demonstrated to me the wealth of research projects available to a teacher of mathematics - he is a true working mathematician (they may retire but they never stop!).

People who have influenced my teaching career

- **Temple H. Fay** from the USM started me on my education career in applied mathematics (and continues to writes papers with me).
- The late **Keith A. Hardie** the pure mathematician from the UCT who had amazing applied mathematics ideas (and published some with me).
- **Igor Fedotov** who started working with me at TUT in 2009 (and started the weekly colloquiums at TUT).
- **Michael Y. Shatalov** formerly from the CSIR and now at TUT who introduced me to modern gyroscopes (my coauthor on a book in production about gyroscopes).
- **Joseph Bohm** who always had something new and exciting together with some real mathematics to talk about (and plays a great game of tennis).
- **Carl Leinbach** who demonstrated to me the wealth of research projects available to a teacher of mathematics - he is a true working mathematician (they may retire but they never stop!).
- **Carsten Schmidt** who taught me to do my teaching in the CAS laboratory whenever possible (I still do this today during my one-day-a week class).

My career before TIME

- I matriculated in 1966 at Clapham High School in Pretoria, South Africa. I worked in my father's automobile workshop during 1967 before enrolling for a BSc in 1969, majoring in mathematics and physics at the University of Pretoria, graduating in 1971.
- After touring Europe for 6 months, I started working as a teacher at Clapham High School in 1972, teaching general science, even though I was busy with an honours degree in mathematics at that time.
- In 1975 I commenced my career at Tshwane University of Technology (TUT), teaching undergraduate physics, moving to the Department of Mathematics and Statistics (DMS) in 1981 and retiring there as "Professor Emeritus" in 2014 (**40 years of service and still keeping my hand in**).
- I received my DSc (Rings and R-modules) in 1984 from the University of Pretoria. Because TUT, as a university of technology (UOT), places a strong emphasis on the education of technologists, I was a "fish out of water".

My career before TIME

- I matriculated in 1966 at Clapham High School in Pretoria, South Africa. I worked in my father's automobile workshop during 1967 before enrolling for a BSc in 1969, majoring in mathematics and physics at the University of Pretoria, graduating in 1971.

- After touring Europe for 6 months, I started working as a teacher at Clapham High School in 1972, teaching general science, even though I was busy with an honours degree in mathematics at that time.

- In 1975 I commenced my career at Tshwane University of Technology (TUT), teaching undergraduate physics, moving to the Department of Mathematics and Statistics (DMS) in 1981 and retiring there as "Professor Emeritus" in 2014 (**40 years of service and still keeping my hand in**).

- I received my DSc (Rings and R-modules) in 1984 from the University of Pretoria. Because TUT, as a university of technology (UOT), places a strong emphasis on the education of technologists, I was a "fish out of water".

My career before TIME

- I matriculated in 1966 at Clapham High School in Pretoria, South Africa. I worked in my father's automobile workshop during 1967 before enrolling for a BSc in 1969, majoring in mathematics and physics at the University of Pretoria, graduating in 1971.

- After touring Europe for 6 months, I started working as a teacher at Clapham High School in 1972, teaching general science, even though I was busy with an honours degree in mathematics at that time.

- In 1975 I commenced my career at Tshwane University of Technology (TUT), teaching undergraduate physics, moving to the Department of Mathematics and Statistics (DMS) in 1981 and retiring there as "Professor Emeritus" in 2014 (**40 years of service and still keeping my hand in**).

- I received my DSc (Rings and R-modules) in 1984 from the University of Pretoria. Because TUT, as a university of technology (UOT), places a strong emphasis on the education of technologists, I was a "fish out of water".

My career before TIME

- I matriculated in 1966 at Clapham High School in Pretoria, South Africa. I worked in my father's automobile workshop during 1967 before enrolling for a BSc in 1969, majoring in mathematics and physics at the University of Pretoria, graduating in 1971.
- After touring Europe for 6 months, I started working as a teacher at Clapham High School in 1972, teaching general science, even though I was busy with an honours degree in mathematics at that time.
- In 1975 I commenced my career at Tshwane University of Technology (TUT), teaching undergraduate physics, moving to the Department of Mathematics and Statistics (DMS) in 1981 and retiring there as "Professor Emeritus" in 2014 (**40 years of service and still keeping my hand in**).
- I received my DSc (Rings and R-modules) in 1984 from the University of Pretoria. Because TUT, as a university of technology (UOT), places a strong emphasis on the education of technologists, I was a "fish out of water".

My career before TIME (continued)

- By 1981, I had already been promoted to chairman of the Department of Mathematics and Statistics (DMS) at TUT, and thus I did not plan to leave TUT for an academic university.
- I decided rather to make a career out of teaching using technology and starting a research programme at TUT that would involve my colleagues.
- My colleagues mostly had four-year bachelor's degrees (or an equivalent degree) and so a research environment in the DMS was almost non-existent.
- I developed a strategy to foster research and human resource development simultaneously as follows:
- With Temple H. Fay from the University of Southern Mississippi (USM), a cooperation agreement was signed between the TUT and USM.

My career before TIME (continued)

- By 1981, I had already been promoted to chairman of the Department of Mathematics and Statistics (DMS) at TUT, and thus I did not plan to leave TUT for an academic university.
- I decided rather to make a career out of teaching using technology and starting a research programme at TUT that would involve my colleagues.
- My colleagues mostly had four-year bachelor's degrees (or an equivalent degree) and so a research environment in the DMS was almost non-existent.
- I developed a strategy to foster research and human resource development simultaneously as follows:
- With Temple H. Fay from the University of Southern Mississippi (USM), a cooperation agreement was signed between the TUT and USM.

My career before TIME (continued)

- By 1981, I had already been promoted to chairman of the Department of Mathematics and Statistics (DMS) at TUT, and thus I did not plan to leave TUT for an academic university.
- I decided rather to make a career out of teaching using technology and starting a research programme at TUT that would involve my colleagues.
- My colleagues mostly had four-year bachelor's degrees (or an equivalent degree) and so a research environment in the DMS was almost non-existent.
- I developed a strategy to foster research and human resource development simultaneously as follows:
- With Temple H. Fay from the University of Southern Mississippi (USM), a cooperation agreement was signed between the TUT and USM.

My career before TIME (continued)

- By 1981, I had already been promoted to chairman of the Department of Mathematics and Statistics (DMS) at TUT, and thus I did not plan to leave TUT for an academic university.
- I decided rather to make a career out of teaching using technology and starting a research programme at TUT that would involve my colleagues.
- My colleagues mostly had four-year bachelor's degrees (or an equivalent degree) and so a research environment in the DMS was almost non-existent.
- I developed a strategy to foster research and human resource development simultaneously as follows:
- With Temple H. Fay from the University of Southern Mississippi (USM), a cooperation agreement was signed between the TUT and USM.

My career before TIME (continued)

- By 1981, I had already been promoted to chairman of the Department of Mathematics and Statistics (DMS) at TUT, and thus I did not plan to leave TUT for an academic university.
- I decided rather to make a career out of teaching using technology and starting a research programme at TUT that would involve my colleagues.
- My colleagues mostly had four-year bachelor's degrees (or an equivalent degree) and so a research environment in the DMS was almost non-existent.
- I developed a strategy to foster research and human resource development simultaneously as follows:
- With Temple H. Fay from the University of Southern Mississippi (USM), a cooperation agreement was signed between the TUT and USM.

My career before TIME (continued)

- In 1995, Temple Fay and James Caveny started teaching USM's master's degree on the Pretoria campus of TUT to four TUT staff members (including me) and five colleagues from various UOT.
- We chose research problems based on what we were familiar with, namely teaching technical undergraduate calculus to departments serving chemistry, engineering, agriculture, nature conservation, et cetera, students.
- In this way we gradually extend our knowledge of what we were teaching to research problems based on what our colleagues and students from other departments at TUT and the South African CSIR were doing
- At the turn of the century TUT introduced its own masters and doctoral programmes based on the USM model.
- Equipped with our newly acquired knowledge, my colleagues and I started publishing in "Mathematics Education" and other technical journals as well as attending conferences (education and technical).

My career before TIME (continued)

- In 1995, Temple Fay and James Caveny started teaching USM's master's degree on the Pretoria campus of TUT to four TUT staff members (including me) and five colleagues from various UOT.
- We chose research problems based on what we were familiar with, namely teaching technical undergraduate calculus to departments serving chemistry, engineering, agriculture, nature conservation, et cetera, students.
- In this way we gradually extend our knowledge of what we were teaching to research problems based on what our colleagues and students from other departments at TUT and the South African CSIR were doing
- At the turn of the century TUT introduced its own masters and doctoral programmes based on the USM model.
- Equipped with our newly acquired knowledge, my colleagues and I started publishing in "Mathematics Education" and other technical journals as well as attending conferences (education and technical).

My career before TIME (continued)

- In 1995, Temple Fay and James Caveny started teaching USM's master's degree on the Pretoria campus of TUT to four TUT staff members (including me) and five colleagues from various UOT.
- We chose research problems based on what we were familiar with, namely teaching technical undergraduate calculus to departments serving chemistry, engineering, agriculture, nature conservation, et cetera, students.
- In this way we gradually extend our knowledge of what we were teaching to research problems based on what our colleagues and students from other departments at TUT and the South African CSIR were doing
- At the turn of the century TUT introduced its own masters and doctoral programmes based on the USM model.
- Equipped with our newly acquired knowledge, my colleagues and I started publishing in "Mathematics Education" and other technical journals as well as attending conferences (education and technical).

My career before TIME (continued)

- In 1995, Temple Fay and James Caveny started teaching USM's master's degree on the Pretoria campus of TUT to four TUT staff members (including me) and five colleagues from various UOT.
- We chose research problems based on what we were familiar with, namely teaching technical undergraduate calculus to departments serving chemistry, engineering, agriculture, nature conservation, et cetera, students.
- In this way we gradually extend our knowledge of what we were teaching to research problems based on what our colleagues and students from other departments at TUT and the South African CSIR were doing
- At the turn of the century TUT introduced its own masters and doctoral programmes based on the USM model.
- Equipped with our newly acquired knowledge, my colleagues and I started publishing in "Mathematics Education" and other technical journals as well as attending conferences (education and technical).

My career before TIME (continued)

- In 1995, Temple Fay and James Caveny started teaching USM's master's degree on the Pretoria campus of TUT to four TUT staff members (including me) and five colleagues from various UOT.
- We chose research problems based on what we were familiar with, namely teaching technical undergraduate calculus to departments serving chemistry, engineering, agriculture, nature conservation, et cetera, students.
- In this way we gradually extend our knowledge of what we were teaching to research problems based on what our colleagues and students from other departments at TUT and the South African CSIR were doing
- At the turn of the century TUT introduced its own masters and doctoral programmes based on the USM model.
- Equipped with our newly acquired knowledge, my colleagues and I started publishing in "Mathematics Education" and other technical journals as well as attending conferences (education and technical).

My career before TIME (continued)

- From 1995 to date, this strategy has resulted in twelve doctoral and more than thirty masters graduates and a host of journal publications. By the year 2002 the original group of twelve including Temple and me had over 35 publications in journals accredited by the South African Education Department.
- In 1997, as a first step towards doing some research in applied mathematics **I advertised my willingness to help any colleagues** who needed mathematical modeling for their own research.
- I was soon approached to give a series of lectures as part of an undergraduate course in explosive technology where I rubbed shoulders with a colleague (in the Department of Explosives Technology) doing her masters degree.
- She was attempting to design and mass produce "friction tiles" **with a prescribed friction coefficient** and was seeking a consistent manufacturing protocol.

My career before TIME (continued)

- From 1995 to date, this strategy has resulted in twelve doctoral and more than thirty masters graduates and a host of journal publications. By the year 2002 the original group of twelve including Temple and me had over 35 publications in journals accredited by the South African Education Department.
- In 1997, as a first step towards doing some research in applied mathematics **I advertised my willingness to help any colleagues** who needed mathematical modeling for their own research.
- I was soon approached to give a series of lectures as part of an undergraduate course in explosive technology where I rubbed shoulders with a colleague (in the Department of Explosives Technology) doing her masters degree.
- She was attempting to design and mass produce "friction tiles" **with a prescribed friction coefficient** and was seeking a consistent manufacturing protocol.

My career before TIME (continued)

- From 1995 to date, this strategy has resulted in twelve doctoral and more than thirty masters graduates and a host of journal publications. By the year 2002 the original group of twelve including Temple and me had over 35 publications in journals accredited by the South African Education Department.
- In 1997, as a first step towards doing some research in applied mathematics **I advertised my willingness to help any colleagues** who needed mathematical modeling for their own research.
- I was soon approached to give a series of lectures as part of an undergraduate course in explosive technology where I rubbed shoulders with a colleague (in the Department of Explosives Technology) doing her masters degree.
- She was attempting to design and mass produce "friction tiles" **with a prescribed friction coefficient** and was seeking a consistent manufacturing protocol.

My career before TIME (continued)

- From 1995 to date, this strategy has resulted in twelve doctoral and more than thirty masters graduates and a host of journal publications. By the year 2002 the original group of twelve including Temple and me had over 35 publications in journals accredited by the South African Education Department.
- In 1997, as a first step towards doing some research in applied mathematics **I advertised my willingness to help any colleagues** who needed mathematical modeling for their own research.
- I was soon approached to give a series of lectures as part of an undergraduate course in explosive technology where I rubbed shoulders with a colleague (in the Department of Explosives Technology) doing her masters degree.
- She was attempting to design and mass produce "friction tiles" **with a prescribed friction coefficient** and was seeking a consistent manufacturing protocol.

International Conference on Mathematics for Industry, Harare, Zimbabwe, 1998

- **Equipment needed:** Any CAS

- **Equipment needed:** Any CAS
- Small ceramic tiles were to be manufactured. The scale of these tiles is illustrated in the next photo:

International Conference on Mathematics for Industry, Harare, Zimbabwe, 1998

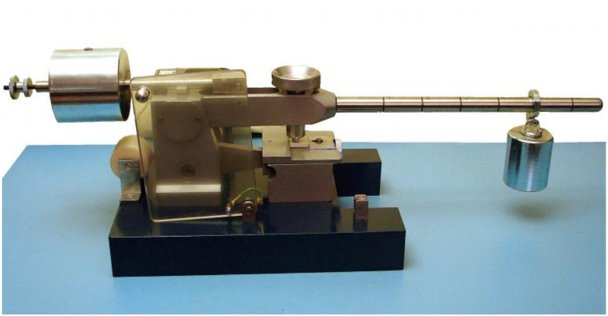
- **Equipment needed:** Any CAS
- Small ceramic tiles were to be manufactured. The scale of these tiles is illustrated in the next photo:
- 

BAM friction meter

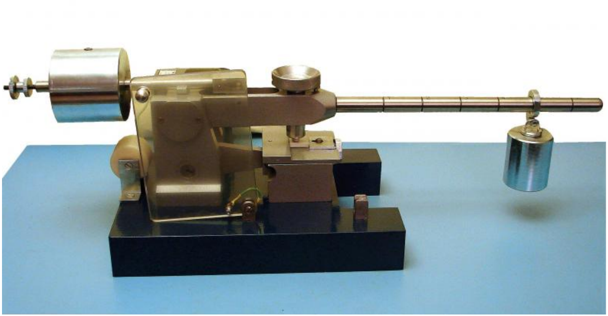
- These tiles were designed to be used in the **B**undesanstalt fur **M**aterialprufung or **BAM** friction meter.

- A small amount of explosive is spread onto the rough surface of the tile and a blunt pin is dragged across the rough surface. The amount of force needed to ignite the explosive can be varied as illustrated above.

BAM friction meter

- These tiles were designed to be used in the **B**undesanstalt fur **M**aterialprufung or **BAM** friction meter.
- 
- A small amount of explosive is spread onto the rough surface of the tile and a blunt pin is dragged across the rough surface. The amount of force needed to ignite the explosive can be varied as illustrated above.

BAM friction meter

- These tiles were designed to be used in the **B**undesanstalt fur **M**aterialprufung or **BAM** friction meter.
- 
- A small amount of explosive is spread onto the rough surface of the tile and a blunt pin is dragged across the rough surface. The amount of force needed to ignite the explosive can be varied as illustrated above.

Design of ceramic tiles

- To manufacture tiles for the South African explosives market with a prescribed **mean roughness value** $R_a = 18.5\ \mu m$, she used ceramic clay and extruded tiles with flat surfaces.

- Then, using a drill bit of radius say $r = 180\ \mu m$, she produced parallel grooves on one surface before baking the tiles.

Design of ceramic tiles

- To manufacture tiles for the South African explosives market with a prescribed **mean roughness value** $R_a = 18.5\ \mu m$, she used ceramic clay and extruded tiles with flat surfaces.

- Then, using a drill bit of radius say $r = 180\ \mu m$, she produced parallel grooves on one surface before baking the tiles.

Design of ceramic tiles

- To manufacture tiles for the South African explosives market with a prescribed **mean roughness value** $R_a = 18.5\ \mu m$, she used ceramic clay and extruded tiles with flat surfaces.

- Then, using a drill bit of radius say $r = 180\ \mu m$, she produced parallel grooves on one surface before baking the tiles.

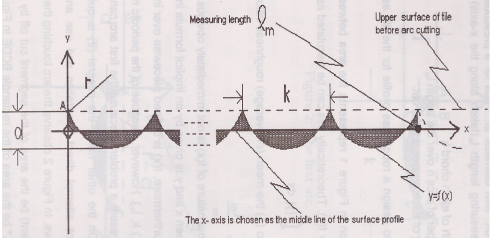
- She needed a model that would **predict** the number of grooves n to milled by a drill bit to produce cross-sectional chords of length $k\ \mu m$ apart to a depth $d\ \mu m$.The proposed surface will be similar to:

Design of ceramic tiles

- To manufacture tiles for the South African explosives market with a prescribed **mean roughness value** $R_a = 18.5\ \mu m$, she used ceramic clay and extruded tiles with flat surfaces.

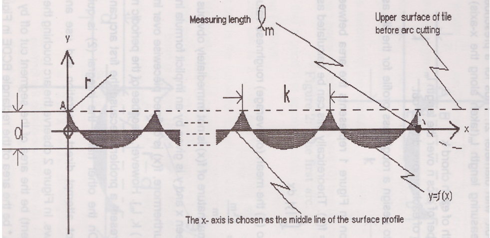
- Then, using a drill bit of radius say $r = 180\ \mu m$, she produced parallel grooves on one surface before baking the tiles.

- She needed a model that would **predict** the number of grooves n to milled by a drill bit to produce cross-sectional chords of length $k\ \mu m$ apart to a depth $d\ \mu m$.The proposed surface will be similar to:

- 

Design of ceramic tiles (continued)

- The function $f(x)$ models the surface displacement above the $x-$axis.



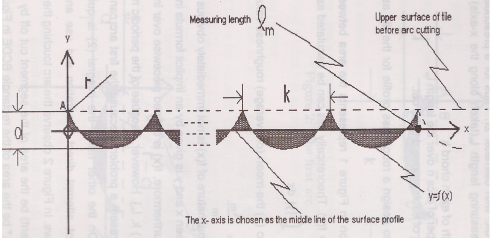
- The **arithmetic mean roughness value** (ISO/DIS 4768/1) of a surface is defined as

$$R_a = \frac{\int_0^{\ell_m} |f(x)|\, dx}{\ell_m}$$

where ℓ_m is the so-called measuring length.

Design of ceramic tiles (continued)

- The function $f(x)$ models the surface displacement above the $x-$axis.



- The **arithmetic mean roughness value** (ISO/DIS 4768/1) of a surface is defined as

$$R_a = \frac{\int_0^{\ell_m} |f(x)|\, dx}{\ell_m}$$

where ℓ_m is the so-called measuring length.

Design of ceramic tiles (continued)

- Prescribing that $r = 180\ \mu m$ and $R_a = 18.5\ \mu m$ and solving a transcendental equation using DERIVE yielded

$$k = 263.9\ \mu m.$$

$$\#12:\ \text{ITERATES}\left[k - \frac{\frac{1}{2}\,180^2\left[2\,\text{ASIN}\left[\frac{k}{2\ 180}\right] - \text{SIN}\left[2\,\text{ASIN}\left[\frac{k}{2\ 180}\right]\right]\right] - k\ 18.5}{\frac{d}{dk}\left[\frac{1}{2}\,180^2\left[2\,\text{ASIN}\left[\frac{k}{2\ 180}\right] - \text{SIN}\left[2\,\text{ASIN}\left[\frac{k}{2\ 180}\right]\right]\right] - k\ 18.\right.}\right.$$

```
#13: [264, 263.883, 263.882, 263.882, 263.882, 263.882, 263.882]

COMMAND: Author Build Calculus Declare Expand Factor Help Jump soLve Manage
         Options Plot Quit Remove Simplify Transfer Unremove moVe Window approX
Enter option
Approx(#12)          C:\DERIVE3.05\CERAM  Free:84%            Derive Algebra
```

- With $\ell_m = 800\ \mu m$ the number of grooves to cut was

$$n = 3$$

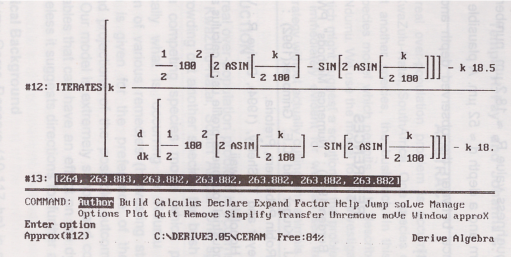
to a depth of

$$d = 58\ \mu m.$$

Design of ceramic tiles (continued)

- Prescribing that $r = 180\ \mu m$ and $R_a = 18.5\ \mu m$ and solving a transcendental equation using DERIVE yielded

$$k = 263.9\ \mu m.$$



- With $\ell_m = 800\ \mu m$ the number of grooves to cut was

$$n = 3$$

to a depth of

$$d = 58\ \mu m.$$

Spin-offs

- One TUT masters degree for a colleague at TUT and twelve USM masters degrees for TUT and other UOT colleagues.
- I obtained **my first applied mathematics publication** in the Harare conference proceedings published in the ISI accredited International Journal of Management and Systems.
- To date, the strategy of encouraging my colleagues and other students to **publish and improve their qualifications** has resulted in
  - twelve doctoral and
  - more than thirty masters graduates with
  - a huge amount of publications
- From 1995 to 2002 there were over 35 publications from the team members in journals accredited by the South African Education Department - we stopped counting then.

Spin-offs

- One TUT masters degree for a colleague at TUT and twelve USM masters degrees for TUT and other UOT colleagues.
- I obtained **my first applied mathematics publication** in the Harare conference proceedings published in the ISI accredited International Journal of Management and Systems.
- To date, the strategy of encouraging my colleagues and other students to **publish and improve their qualifications** has resulted in
  - twelve doctoral and
  - more than thirty masters graduates with
  - a huge amount of publications
- From 1995 to 2002 there were over 35 publications from the team members in journals accredited by the South African Education Department - we stopped counting then.

Spin-offs

- One TUT masters degree for a colleague at TUT and twelve USM masters degrees for TUT and other UOT colleagues.
- I obtained **my first applied mathematics publication** in the Harare conference proceedings published in the ISI accredited International Journal of Management and Systems.
- To date, the strategy of encouraging my colleagues and other students to **publish and improve their qualifications** has resulted in
  - twelve doctoral and
  - more than thirty masters graduates with
  - a huge amount of publications
- From 1995 to 2002 there were over 35 publications from the team members in journals accredited by the South African Education Department - we stopped counting then.

Spin-offs

- One TUT masters degree for a colleague at TUT and twelve USM masters degrees for TUT and other UOT colleagues.
- I obtained **my first applied mathematics publication** in the Harare conference proceedings published in the ISI accredited International Journal of Management and Systems.
- To date, the strategy of encouraging my colleagues and other students to **publish and improve their qualifications** has resulted in
  - twelve doctoral and
  - more than thirty masters graduates with
  - a huge amount of publications
- From 1995 to 2002 there were over 35 publications from the team members in journals accredited by the South African Education Department - we stopped counting then.

Spin-offs

- One TUT masters degree for a colleague at TUT and twelve USM masters degrees for TUT and other UOT colleagues.
- I obtained **my first applied mathematics publication** in the Harare conference proceedings published in the ISI accredited International Journal of Management and Systems.
- To date, the strategy of encouraging my colleagues and other students to **publish and improve their qualifications** has resulted in
  - twelve doctoral and
  - more than thirty masters graduates with
  - a huge amount of publications
- From 1995 to 2002 there were over 35 publications from the team members in journals accredited by the South African Education Department - we stopped counting then.

Spin-offs

- One TUT masters degree for a colleague at TUT and twelve USM masters degrees for TUT and other UOT colleagues.
- I obtained **my first applied mathematics publication** in the Harare conference proceedings published in the ISI accredited International Journal of Management and Systems.
- To date, the strategy of encouraging my colleagues and other students to **publish and improve their qualifications** has resulted in
  - twelve doctoral and
  - more than thirty masters graduates with
  - a huge amount of publications
- From 1995 to 2002 there were over 35 publications from the team members in journals accredited by the South African Education Department - we stopped counting then.

Spin-offs

- One TUT masters degree for a colleague at TUT and twelve USM masters degrees for TUT and other UOT colleagues.
- I obtained **my first applied mathematics publication** in the Harare conference proceedings published in the ISI accredited International Journal of Management and Systems.
- To date, the strategy of encouraging my colleagues and other students to **publish and improve their qualifications** has resulted in
  - twelve doctoral and
  - more than thirty masters graduates with
  - a huge amount of publications
- From 1995 to 2002 there were over 35 publications from the team members in journals accredited by the South African Education Department - we stopped counting then.

My transition to TIME (continued)

- I was introduced to TIME conferences in 2002 after attending the 2^{nd} International Conference on Teaching Mathematics in Crete (ICTM) .
- I have attended every TIME conference since 2002 and will recall the papers that I presented (mainly at) TIME and other conferences as well as the spin-offs that occurred due to these papers.
- First I will discuss an interesting classroom experiment with a non-trivial twist to it.

My transition to TIME (continued)

- I was introduced to TIME conferences in 2002 after attending the 2^{nd} International Conference on Teaching Mathematics in Crete (ICTM) .
- I have attended every TIME conference since 2002 and will recall the papers that I presented (mainly at) TIME and other conferences as well as the spin-offs that occurred due to these papers.
- First I will discuss an interesting classroom experiment with a non-trivial twist to it.

My transition to TIME (continued)

- I was introduced to TIME conferences in 2002 after attending the 2^{nd} International Conference on Teaching Mathematics in Crete (ICTM) .
- I have attended every TIME conference since 2002 and will recall the papers that I presented (mainly at) TIME and other conferences as well as the spin-offs that occurred due to these papers.
- First I will discuss an interesting classroom experiment with a non-trivial twist to it.

Second ICTM, Crete 2002: The bouncing soccer ball (with Anton van Wyk)

- **Equipment needed:**
  - Plastic soccer ball
  - Kitchen or chemical scale
  - Bathroom scale
  - Ruler
  - Measuring tape or metre rule
  - Any CAS

- Typically our students have two semesters of calculus before they register for an introductory ODE course.

- Once the concept of second order ODEs with constant coefficients has been reached and a few easy exercises have been done, we introduce the idea of a spring-damper pair, with the accompanying concepts of over damped, under damped and critically damped free motion.

Second ICTM, Crete 2002: The bouncing soccer ball (with Anton van Wyk)

- **Equipment needed:**
    - Plastic soccer ball
    - Kitchen or chemical scale
    - Bathroom scale
    - Ruler
    - Measuring tape or metre rule
    - Any CAS

- Typically our students have two semesters of calculus before they register for an introductory ODE course.

- Once the concept of second order ODEs with constant coefficients has been reached and a few easy exercises have been done, we introduce the idea of a spring-damper pair, with the accompanying concepts of over damped, under damped and critically damped free motion.

Second ICTM, Crete 2002: The bouncing soccer ball (with Anton van Wyk)

- **Equipment needed:**
  - Plastic soccer ball
  - Kitchen or chemical scale
  - Bathroom scale
  - Ruler
  - Measuring tape or metre rule
  - Any CAS
- Typically our students have two semesters of calculus before they register for an introductory ODE course.
- Once the concept of second order ODEs with constant coefficients has been reached and a few easy exercises have been done, we introduce the idea of a spring-damper pair, with the accompanying concepts of over damped, under damped and critically damped free motion.

Second ICTM, Crete 2002: The bouncing soccer ball (with Anton van Wyk)

- **Equipment needed:**
  - Plastic soccer ball
  - Kitchen or chemical scale
  - Bathroom scale
  - Ruler
  - Measuring tape or metre rule
  - Any CAS
- Typically our students have two semesters of calculus before they register for an introductory ODE course.
- Once the concept of second order ODEs with constant coefficients has been reached and a few easy exercises have been done, we introduce the idea of a spring-damper pair, with the accompanying concepts of over damped, under damped and critically damped free motion.

Second ICTM, Crete 2002: The bouncing soccer ball (with Anton van Wyk)

- **Equipment needed:**
  - Plastic soccer ball
  - Kitchen or chemical scale
  - Bathroom scale
  - Ruler
  - Measuring tape or metre rule
  - Any CAS
- Typically our students have two semesters of calculus before they register for an introductory ODE course.
- Once the concept of second order ODEs with constant coefficients has been reached and a few easy exercises have been done, we introduce the idea of a spring-damper pair, with the accompanying concepts of over damped, under damped and critically damped free motion.

Second ICTM, Crete 2002: The bouncing soccer ball (with Anton van Wyk)

- **Equipment needed:**
  - Plastic soccer ball
  - Kitchen or chemical scale
  - Bathroom scale
  - Ruler
  - Measuring tape or metre rule
  - Any CAS
- Typically our students have two semesters of calculus before they register for an introductory ODE course.
- Once the concept of second order ODEs with constant coefficients has been reached and a few easy exercises have been done, we introduce the idea of a spring-damper pair, with the accompanying concepts of over damped, under damped and critically damped free motion.

Second ICTM, Crete 2002: The bouncing soccer ball (with Anton van Wyk)

- **Equipment needed:**
  - Plastic soccer ball
  - Kitchen or chemical scale
  - Bathroom scale
  - Ruler
  - Measuring tape or metre rule
  - Any CAS
- Typically our students have two semesters of calculus before they register for an introductory ODE course.
- Once the concept of second order ODEs with constant coefficients has been reached and a few easy exercises have been done, we introduce the idea of a spring-damper pair, with the accompanying concepts of over damped, under damped and critically damped free motion.

Second ICTM, Crete 2002: The bouncing soccer ball (with Anton van Wyk)

- **Equipment needed:**
  - Plastic soccer ball
  - Kitchen or chemical scale
  - Bathroom scale
  - Ruler
  - Measuring tape or metre rule
  - Any CAS
- Typically our students have two semesters of calculus before they register for an introductory ODE course.
- Once the concept of second order ODEs with constant coefficients has been reached and a few easy exercises have been done, we introduce the idea of a spring-damper pair, with the accompanying concepts of over damped, under damped and critically damped free motion.

Second ICTM, Crete 2002: The bouncing soccer ball (with Anton van Wyk)

- **Equipment needed:**
  - Plastic soccer ball
  - Kitchen or chemical scale
  - Bathroom scale
  - Ruler
  - Measuring tape or metre rule
  - Any CAS

- Typically our students have two semesters of calculus before they register for an introductory ODE course.

- Once the concept of second order ODEs with constant coefficients has been reached and a few easy exercises have been done, we introduce the idea of a spring-damper pair, with the accompanying concepts of over damped, under damped and critically damped free motion.

The bouncing soccer ball

- During this period of conceptual development, we give small groups of students a soccer ball. Initially they play with it before we start some measurements.

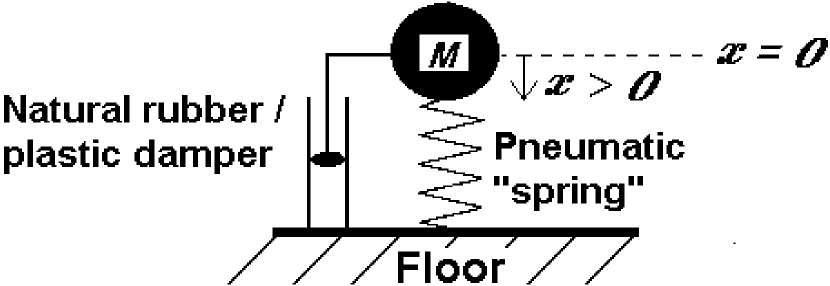
The bouncing soccer ball

- During this period of conceptual development, we give small groups of students a soccer ball. Initially they play with it before we start some measurements.
- 

The bouncing soccer ball (continued)

- The ODE to consider solving for the simplified model of one bounce is

$$M\ddot{x} + B\dot{x} + Kx = 0$$

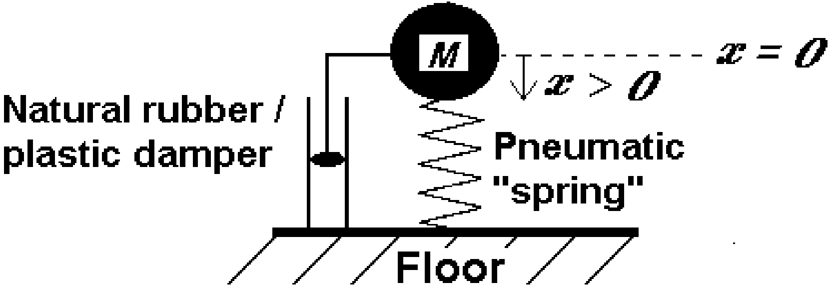


- A **kitchen scale** is used to measure the mass of the ball M.
- To measure the spring constant K, place the ball on top of a **bathroom scale**.
  - Push down and hold the position at say a reading of 20 kg on the scale.
  - Measure the ball compression with a **ruler**.

The bouncing soccer ball (continued)

- The ODE to consider solving for the simplified model of one bounce is

$$M\ddot{x} + B\dot{x} + Kx = 0$$

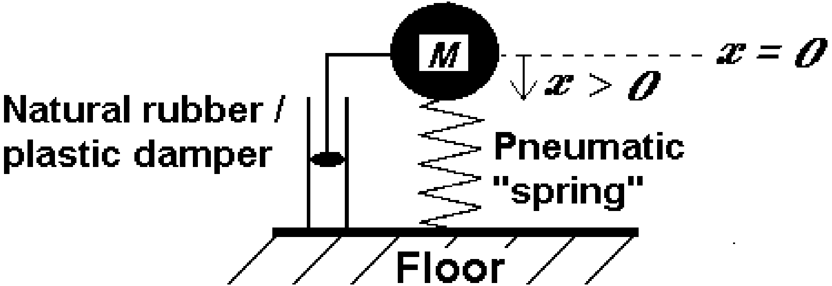


- A **kitchen scale** is used to measure the mass of the ball M.
- To measure the spring constant K, place the ball on top of a **bathroom scale**.
  - Push down and hold the position at say a reading of 20 kg on the scale.
  - Measure the ball compression with a **ruler**.

The bouncing soccer ball (continued)

- The ODE to consider solving for the simplified model of one bounce is

$$M\ddot{x} + B\dot{x} + Kx = 0$$

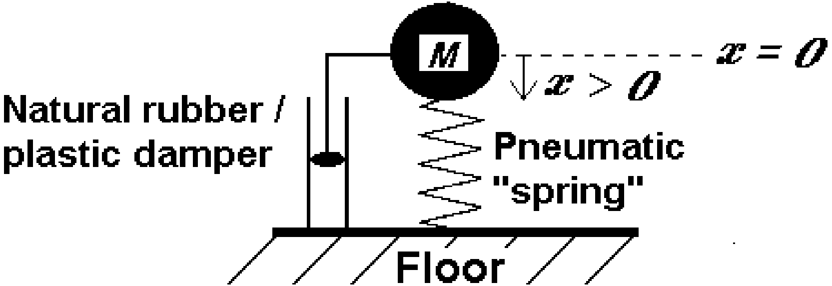


- A **kitchen scale** is used to measure the mass of the ball M.
- To measure the spring constant K, place the ball on top of a **bathroom scale**.
  - Push down and hold the position at say a reading of 20 kg on the scale.
  - Measure the ball compression with a **ruler**.

The bouncing soccer ball (continued)

- The ODE to consider solving for the simplified model of one bounce is

$$M\ddot{x} + B\dot{x} + Kx = 0$$

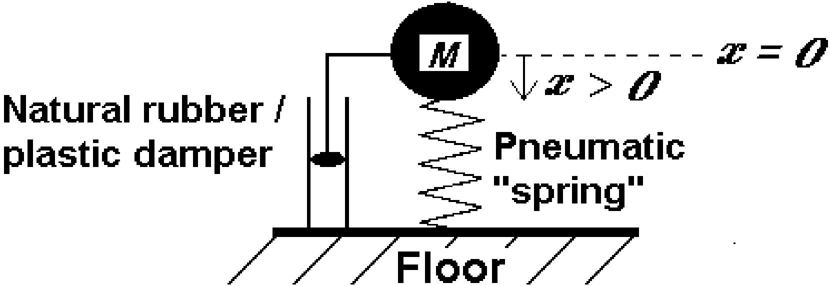


- A **kitchen scale** is used to measure the mass of the ball M.
- To measure the spring constant K, place the ball on top of a **bathroom scale**.
  - Push down and hold the position at say a reading of 20 kg on the scale.
  - Measure the ball compression with a **ruler**.

The bouncing soccer ball (continued)

- The ODE to consider solving for the simplified model of one bounce is

$$M\ddot{x} + B\dot{x} + Kx = 0$$



- A **kitchen scale** is used to measure the mass of the ball M.
- To measure the spring constant K, place the ball on top of a **bathroom scale**.
  - Push down and hold the position at say a reading of 20 kg on the scale.
  - Measure the ball compression with a **ruler**.

The bouncing soccer ball (continued)

- We found the compression distance to be approximately $\Delta x = 6\,\mathrm{cm}$.
- From Hooke's law the ball "spring constant" k may be obtained.

$$K = \frac{Mg}{\Delta x}$$
$$K = \frac{20 \times 9.8}{0.06} \approx 3.3 \times 10^3\,\mathrm{N\,m^{-1}}$$

The bouncing soccer ball (continued)

- 

- We found the compression distance to be approximately $\Delta x = 6\,\mathrm{cm}$.
- From Hooke's law the ball "spring constant" k may be obtained.

$$K = \frac{Mg}{\Delta x}$$

$$K = \frac{20 \times 9.8}{0.06}$$

$$\approx 3.3 \times 10^3\,\mathrm{N\,m^{-1}}$$

The bouncing soccer ball (continued)

- 

- We found the compression distance to be approximately $\Delta x = 6\,\mathrm{cm}$.
- From Hooke's law the ball "spring constant" k may be obtained.

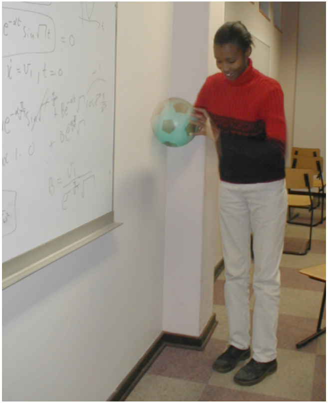
$$
\begin{aligned}
K &= \frac{Mg}{\Delta x} \\
K &= \frac{20 \times 9.8}{0.06} \\
&\approx 3.3 \times 10^{3}\,\mathrm{N\,m^{-1}}
\end{aligned}
$$

The bouncing soccer ball (continued)

- Determining the damping coefficient B of the rubber ball is more difficult. One cannot pull on the ball as one would on a shock absorber.

The bouncing soccer ball (continued)

- With a little mathematical "trickery" one can use the **coefficient of restitution** of the ball instead, by measuring the height of drop h_1 and the height of bounce h_2 using a **tape measure**.



Coefficient of restitution

- The coefficient of restitution R of the ball is the ratio of the incident velocity before bounce to the reflection velocity after bounce.
- The high school formula
$$v^2 = 2gh$$
is used to show that
-
$$R = \sqrt{\frac{h_2}{h_1}} \approx 0.62$$
and this can be used to determine the damping coefficient B.
- Applying these measurements to the equation of motion of the ball considered as a spring-damper pair, the trajectory of the ball may be modelled mathematically using DERIVE or any suitable CAS.

Coefficient of restitution

- The coefficient of restitution R of the ball is the ratio of the incident velocity before bounce to the reflection velocity after bounce.
- The high school formula
$$v^2 = 2gh$$
is used to show that
-
$$\begin{aligned} R &= \sqrt{\frac{h_2}{h_1}} \\ &\approx 0.62 \end{aligned}$$
and this can be used to determine the damping coefficient B.
- Applying these measurements to the equation of motion of the ball considered as a spring-damper pair, the trajectory of the ball may be modelled mathematically using DERIVE or any suitable CAS.

Coefficient of restitution

- The coefficient of restitution R of the ball is the ratio of the incident velocity before bounce to the reflection velocity after bounce.
- The high school formula

$$v^2 = 2gh$$

is used to show that

-

$$\begin{aligned} R &= \sqrt{\frac{h_2}{h_1}} \\ &\approx 0.62 \end{aligned}$$

and this can be used to determine the damping coefficient B.

- Applying these measurements to the equation of motion of the ball considered as a spring-damper pair, the trajectory of the ball may be modelled mathematically using DERIVE or any suitable CAS.

Coefficient of restitution

- The coefficient of restitution R of the ball is the ratio of the incident velocity before bounce to the reflection velocity after bounce.
- The high school formula

$$v^2 = 2gh$$

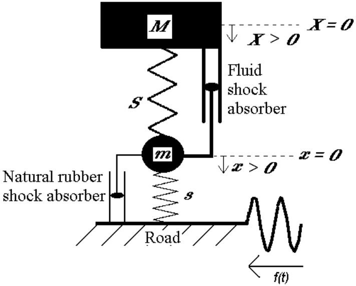
is used to show that

- $$\begin{aligned} R &= \sqrt{\frac{h_2}{h_1}} \\ &\approx 0.62 \end{aligned}$$

and this can be used to determine the damping coefficient B.

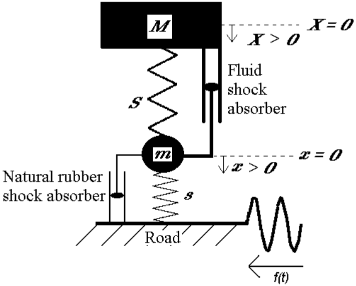
- Applying these measurements to the equation of motion of the ball considered as a spring-damper pair, the trajectory of the ball may be modelled mathematically using DERIVE or any suitable CAS.

New Zealand DELTA03: An affordable, realistic student model of a motor-vehicle suspension system



-

New Zealand DELTA03: An affordable, realistic student model of a motor-vehicle suspension system

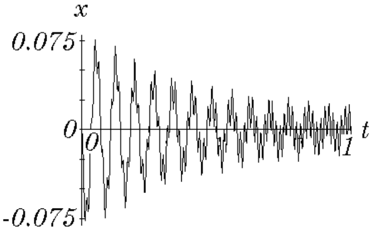


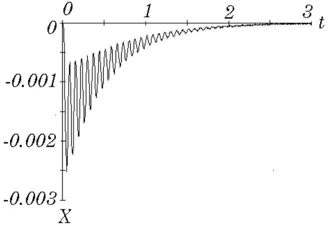
Motor-vehicle suspension system

- $$\begin{aligned} f(t) - kx - \phi\dot{x} + K(X - x) &= m\ddot{x} \\ -\Phi\dot{X} - K(X - x) &= M\ddot{X} \\ x(0) = X(0) = 0 \text{ and } \dot{x}(0) &= \dot{X}(0) = 0 \end{aligned}$$

Motor-vehicle suspension system

- $$\begin{aligned} f(t) - kx - \phi\dot{x} + K(X - x) &= m\ddot{x} \\ -\Phi\dot{X} - K(X - x) &= M\ddot{X} \\ x(0) = X(0) = 0 \text{ and } \dot{x}(0) &= \dot{X}(0) = 0 \end{aligned}$$

- 



Spin-off publications (SAJS 2001, 2004) and SA patent with Keith Hardie and Temple Fay

- I had our team (Temple Fay, Keith Hardie, Anton van Wyk, Ansie Greeff et cetera) thinking about bouncing soccer balls and vehicle suspension systems.
- While driving over a corrugated dirt road in the Pilansberg Game Reserve near Pretoria (close to Buffelspoort where TIME2008 was held), I asked "I wonder how corrugations form". Keith went quiet for a while and then announced "I think I know how".
- Shortly after this the articles "**Dirt road Corrugations**" and "**Towards the interactive wheel**" appeared in the accredited South African Journal of Science and
- a South African patent "**Wheel**" appeared.

Spin-off publications (SAJS 2001, 2004) and SA patent with Keith Hardie and Temple Fay

- I had our team (Temple Fay, Keith Hardie, Anton van Wyk, Ansie Greeff et cetera) thinking about bouncing soccer balls and vehicle suspension systems.
- While driving over a corrugated dirt road in the Pilansberg Game Reserve near Pretoria (close to Buffelspoort where TIME2008 was held), I asked "I wonder how corrugations form". Keith went quiet for a while and then announced "I think I know how".
- Shortly after this the articles "**Dirt road Corrugations**" and "**Towards the interactive wheel**" appeared in the accredited South African Journal of Science and
- a South African patent "**Wheel**" appeared.

Spin-off publications (SAJS 2001, 2004) and SA patent with Keith Hardie and Temple Fay

- I had our team (Temple Fay, Keith Hardie, Anton van Wyk, Ansie Greeff et cetera) thinking about bouncing soccer balls and vehicle suspension systems.
- While driving over a corrugated dirt road in the Pilansberg Game Reserve near Pretoria (close to Buffelspoort where TIME2008 was held), I asked "I wonder how corrugations form". Keith went quiet for a while and then announced "I think I know how".
- Shortly after this the articles "**Dirt road Corrugations**" and "**Towards the interactive wheel**" appeared in the accredited South African Journal of Science and
- a South African patent "**Wheel**" appeared.

Spin-off publications (SAJS 2001, 2004) and SA patent with Keith Hardie and Temple Fay

- I had our team (Temple Fay, Keith Hardie, Anton van Wyk, Ansie Greeff et cetera) thinking about bouncing soccer balls and vehicle suspension systems.
- While driving over a corrugated dirt road in the Pilansberg Game Reserve near Pretoria (close to Buffelspoort where TIME2008 was held), I asked "I wonder how corrugations form". Keith went quiet for a while and then announced "I think I know how".
- Shortly after this the articles "**Dirt road Corrugations**" and "**Towards the interactive wheel**" appeared in the accredited South African Journal of Science and
- a South African patent "**Wheel**" appeared.

Wheel

- A South African patent "**Wheel**" appeared in 2005. Many people ask me about this and I usually answer them: "We invented the square wheel which will not role back on hills".
- **BC discussed** an improvement on the square wheel:

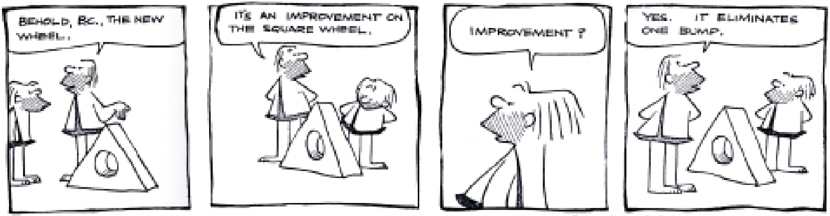
Wheel

- A South African patent "**Wheel**" appeared in 2005. Many people ask me about this and I usually answer them: "We invented the square wheel which will not role back on hills".
- **BC discussed** an improvement on the square wheel:

Wheel

- A South African patent "**Wheel**" appeared in 2005. Many people ask me about this and I usually answer them: "We invented the square wheel which will not role back on hills".

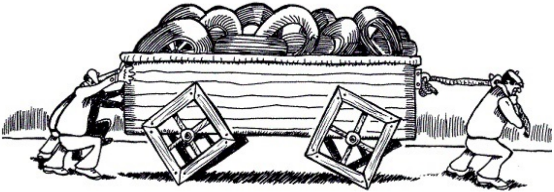
- **BC discussed** an improvement on the square wheel:

- 

By permission of Johnny Hart and Field Enterprises, Inc.

Wheel (continued)

- Stan Wagon improved on BC's wheel by making a square wheel bicycle that rides smoothly over a catenery road:



Wheel (continued)

- **Seriously,** our patent outlines the design of a wheel with a tyre that resists the formation of dirt road corrugations by **reducing the tangential vibrations of the surface of the tyre** on the road. Our contention is that **it is the tangential vibrations** of the tyre and **not the radial vibrations** of the tyre and vehicle springs **that cause corrugations.**

Wheel (continued)

- **Seriously,** our patent outlines the design of a wheel with a tyre that resists the formation of dirt road corrugations by **reducing the tangential vibrations of the surface of the tyre** on the road. Our contention is that **it is the tangential vibrations** of the tyre and **not the radial vibrations** of the tyre and vehicle springs **that cause corrugations.**
- **Human resource development:**

Wheel (continued)

- **Seriously,** our patent outlines the design of a wheel with a tyre that resists the formation of dirt road corrugations by **reducing the tangential vibrations of the surface of the tyre** on the road. Our contention is that **it is the tangential vibrations** of the tyre and **not the radial vibrations** of the tyre and vehicle springs **that cause corrugations.**
- **Human resource development:**
  - I supervised, with Temple Fay, the masters (USM) and doctoral (TUT) studies of Dr. Annatjie Bekker our colleague from the Central UOT in South Africa. Annatjie's thesis dealt with dirt road corrugations and provided evidence to support our patent wheel design.

Wheel (continued)

- **Seriously,** our patent outlines the design of a wheel with a tyre that resists the formation of dirt road corrugations by **reducing the tangential vibrations of the surface of the tyre** on the road. Our contention is that **it is the tangential vibrations** of the tyre and **not the radial vibrations** of the tyre and vehicle springs **that cause corrugations.**
- **Human resource development:**
  - I supervised, with Temple Fay, the masters (USM) and doctoral (TUT) studies of Dr. Annatjie Bekker our colleague from the Central UOT in South Africa. Annatjie's thesis dealt with dirt road corrugations and provided evidence to support our patent wheel design.
  - Indeed, she found among other results that corrugations have a wavelength of about $40\,\mathrm{cm}$ and that the frequency of vibrations (about $42\,\mathrm{Hz}$) of a vehicle travelling at $60\,\mathrm{km\,h^{-1}}$ agrees well with the frequency (about $44\ \mathrm{Hz}$) of tangential vibrations of a rubber tyre while radial spring frequency (about $2\,\mathrm{Hz}$) vibrations are not even of the same order of magnitude as tangential vibrations.

Visit-Me TIME, Vienna, 2002: Dimension Analyses (with Temple Fay)

- Dimension analyses (DA) is folklore, but DA that results in a system of say three simultaneous equations with five unknowns is not readily understood by undergraduates at a UOT.
- The advent of CAS has made such analyses possible for our undergraduates.
- As an example of DA, consider that the period of **small** oscillation T of a bob mass M on a massless rod of length ℓ is known to be

$$T \approx 2\pi\sqrt{\frac{\ell}{g}}.$$

- In order to derive this formula, consider that the period of small oscillation T may depend on the initial angular displacement θ_0, the length of the (massless) rod ℓ, mass of the bob M and gravitational acceleration g.

Visit-Me TIME, Vienna, 2002: Dimension Analyses (with Temple Fay)

- Dimension analyses (DA) is folklore, but DA that results in a system of say three simultaneous equations with five unknowns is not readily understood by undergraduates at a UOT.
- The advent of CAS has made such analyses possible for our undergraduates.
- As an example of DA, consider that the period of **small** oscillation T of a bob mass M on a massless rod of length ℓ is known to be

$$T \approx 2\pi\sqrt{\frac{\ell}{g}}.$$

- In order to derive this formula, consider that the period of small oscillation T may depend on the initial angular displacement θ_0, the length of the (massless) rod ℓ, mass of the bob M and gravitational acceleration g.

Visit-Me TIME, Vienna, 2002: Dimension Analyses (with Temple Fay)

- Dimension analyses (DA) is folklore, but DA that results in a system of say three simultaneous equations with five unknowns is not readily understood by undergraduates at a UOT.
- The advent of CAS has made such analyses possible for our undergraduates.
- As an example of DA, consider that the period of **small** oscillation T of a bob mass M on a massless rod of length ℓ is known to be
$$T \approx 2\pi\sqrt{\frac{\ell}{g}}.$$
- In order to derive this formula, consider that the period of small oscillation T may depend on the initial angular displacement θ_0, the length of the (massless) rod ℓ, mass of the bob M and gravitational acceleration g.

Visit-Me TIME, Vienna, 2002: Dimension Analyses (with Temple Fay)

- Dimension analyses (DA) is folklore, but DA that results in a system of say three simultaneous equations with five unknowns is not readily understood by undergraduates at a UOT.
- The advent of CAS has made such analyses possible for our undergraduates.
- As an example of DA, consider that the period of **small** oscillation T of a bob mass M on a massless rod of length ℓ is known to be

$$T \approx 2\pi\sqrt{\frac{\ell}{g}}.$$

- In order to derive this formula, consider that the period of small oscillation T may depend on the initial angular displacement θ_0, the length of the (massless) rod ℓ, mass of the bob M and gravitational acceleration g.

Dimension Analyses

- In order to derive this result using DA, we form the dimensionless product

$$\Pi_{Gen} = T^{x_1} \times \theta_0^{x_2} \times \ell^{x_3} \times M^{x_4} \times g^{x_5}.$$

- Hence we obtain the system of linear equations:

$$\begin{aligned}
\mathrm{m}: &\quad 0x_1 + 0x_2 + x_3 + 0x_4 + x_5 &= 0 \\
\mathrm{kg}: &\quad 0x_1 + 0x_2 + 0x_3 + x_4 + 0x_5 &= 0 \\
\mathrm{s}: &\quad x_1 + 0x_2 + 0x_3 + 0x_4 - 2x_5 &= 0
\end{aligned}$$

- Solving this system using DERIVE® yields a general solution:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = a \begin{pmatrix} 1 \\ 0 \\ -\frac{1}{2} \\ 0 \\ \frac{1}{2} \end{pmatrix} + b \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad a, b \in \mathbb{R}$$

Dimension Analyses

- In order to derive this result using DA, we form the dimensionless product

$$\Pi_{Gen} = T^{x_1} \times \theta_0^{x_2} \times \ell^{x_3} \times M^{x_4} \times g^{x_5}.$$

- Hence we obtain the system of linear equations:

$$\begin{aligned} \text{m}: \quad & 0x_1 + 0x_2 + x_3 + 0x_4 + x_5 &= 0 \\ \text{kg}: \quad & 0x_1 + 0x_2 + 0x_3 + x_4 + 0x_5 &= 0 \\ \text{s}: \quad & x_1 + 0x_2 + 0x_3 + 0x_4 - 2x_5 &= 0 \end{aligned}$$

- Solving this system using DERIVE® yields a general solution:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = a \begin{pmatrix} 1 \\ 0 \\ -\frac{1}{2} \\ 0 \\ \frac{1}{2} \end{pmatrix} + b \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad a, b \in \mathbb{R}$$

Dimension Analyses

- In order to derive this result using DA, we form the dimensionless product

$$\Pi_{Gen} = T^{x_1} \times \theta_0^{x_2} \times \ell^{x_3} \times M^{x_4} \times g^{x_5}.$$

- Hence we obtain the system of linear equations:

$$\begin{aligned}
\text{m}: \quad & 0x_1 + 0x_2 + x_3 + 0x_4 + x_5 &= 0\\
\text{kg}: \quad & 0x_1 + 0x_2 + 0x_3 + x_4 + 0x_5 &= 0\\
\text{s}: \quad & x_1 + 0x_2 + 0x_3 + 0x_4 - 2x_5 &= 0
\end{aligned}$$

- Solving this system using DERIVE® yields a general solution:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = a \begin{pmatrix} 1 \\ 0 \\ -\frac{1}{2} \\ 0 \\ \frac{1}{2} \end{pmatrix} + b \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad a, b \in \mathbb{R}$$

Fundamental solutions

- Hence we see that $x_4 = 0$ and so mass does not play a role! It is obvious that we have two fundamental solutions:
- $$\begin{pmatrix} 1 \\ 0 \\ -\frac{1}{2} \\ 0 \\ \frac{1}{2} \end{pmatrix} \text{ and } \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$
- This means essentially that we have two non dimensional products Π_1 and Π_2 such that

$$\Pi_1 = T \times (\theta_0)^0 \times \ell^{-1/2} \times m^0 \times g^{1/2} = T\sqrt{\frac{g}{\ell}}$$

and

$$\Pi_2 = T^0 \times (\theta_0)^1 \times \ell^0 \times m^0 \times g^0 = \theta_0$$

Fundamental solutions

- Hence we see that $x_4 = 0$ and so mass does not play a role! It is obvious that we have two fundamental solutions:
-
$$\begin{pmatrix} 1 \\ 0 \\ -\frac{1}{2} \\ 0 \\ \frac{1}{2} \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$
- This means essentially that we have two non dimensional products Π_1 and Π_2 such that

$$\Pi_1 = T \times (\theta_0)^0 \times \ell^{-1/2} \times m^0 \times g^{1/2} = T\sqrt{\frac{g}{\ell}}$$

and

$$\Pi_2 = T^0 \times (\theta_0)^1 \times \ell^0 \times m^0 \times g^0 = \theta_0$$

Fundamental solutions

- Hence we see that $x_4 = 0$ and so mass does not play a role! It is obvious that we have two fundamental solutions:
- $$\begin{pmatrix} 1 \\ 0 \\ -\frac{1}{2} \\ 0 \\ \frac{1}{2} \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$
- This means essentially that we have two non dimensional products Π_1 and Π_2 such that

$$\Pi_1 = T \times (\theta_0)^0 \times \ell^{-1/2} \times m^0 \times g^{1/2} = T\sqrt{\frac{g}{\ell}}$$

and

$$\Pi_2 = T^0 \times (\theta_0)^1 \times \ell^0 \times m^0 \times g^0 = \theta_0$$

Fundamental solutions

- Hence we see that $x_4 = 0$ and so mass does not play a role! It is obvious that we have two fundamental solutions:
- $$\begin{pmatrix} 1 \\ 0 \\ -\frac{1}{2} \\ 0 \\ \frac{1}{2} \end{pmatrix} \text{ and } \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$
- This means essentially that we have two non dimensional products Π_1 and Π_2 such that

$$\Pi_1 = T \times (\theta_0)^0 \times \ell^{-1/2} \times m^0 \times g^{1/2} = T\sqrt{\frac{g}{\ell}}$$

and

$$\Pi_2 = T^0 \times (\theta_0)^1 \times \ell^0 \times m^0 \times g^0 = \theta_0$$

Buckingham's Theorem

- Buckingham's theorem yields

$$T = \frac{\Psi(\theta_0)}{\left(\frac{g}{\ell}\right)^{1/2}} = \Psi(\theta_0) \times \sqrt{\frac{\ell}{g}}$$

- where Ψ is a function of θ_0. An experiment can be conducted using small oscillations that shows that

$$\Psi(\theta_0) \approx 2\pi.$$

- **Spin off publication:** IJMEST (2002). "Dimensional Analyses: An Elegant technique for facilitating the teaching of mathematical modelling" with Temple Fay.

Buckingham's Theorem

- Buckingham's theorem yields

$$T = \frac{\Psi(\theta_0)}{\left(\frac{g}{\ell}\right)^{1/2}} = \Psi(\theta_0) \times \sqrt{\frac{\ell}{g}}$$

- where Ψ is a function of θ_0. An experiment can be conducted using small oscillations that shows that

$$\Psi(\theta_0) \approx 2\pi.$$

- **Spin off publication:** IJMEST (2002). "Dimensional Analyses: An Elegant technique for facilitating the teaching of mathematical modelling" with Temple Fay.

Buckingham's Theorem

- Buckingham's theorem yields

$$T = \frac{\Psi(\theta_0)}{\left(\frac{g}{\ell}\right)^{1/2}} = \Psi(\theta_0) \times \sqrt{\frac{\ell}{g}}$$

- where Ψ is a function of θ_0. An experiment can be conducted using small oscillations that shows that

$$\Psi(\theta_0) \approx 2\pi.$$

- **Spin off publication:** IJMEST (2002). "Dimensional Analyses: An Elegant technique for facilitating the teaching of mathematical modelling" with Temple Fay.

Montreal TIME 2004: Testing the Accuracy of DERIVE's "RK4" Routine (with Temple Fay)

- Here we offer a simple DERIVE procedure that provides a measure of the accuracy of the Runge-Kutta order 4 DERIVE routine "RK4".
- Consider a linear system (although the routine works well with nonlinear systems):

$$\ddot{x} - 2\dot{x} - 3x = -4e^{t}$$
$$x(0) = 2, \quad \dot{x}(0) = 0.$$

- This has exact solution

$$x = e^{-t} + e^{t} = 2\cosh t.$$

Montreal TIME 2004: Testing the Accuracy of DERIVE's "RK4" Routine (with Temple Fay)

- Here we offer a simple DERIVE procedure that provides a measure of the accuracy of the Runge-Kutta order 4 DERIVE routine "RK4".
- Consider a linear system (although the routine works well with nonlinear systems):

$$\ddot{x} - 2\dot{x} - 3x = -4e^t$$
$$x(0) = 2, \quad \dot{x}(0) = 0.$$

- This has exact solution

$$x = e^{-t} + e^t = 2\cosh t.$$

Montreal TIME 2004: Testing the Accuracy of DERIVE's "RK4" Routine (with Temple Fay)

- Here we offer a simple DERIVE procedure that provides a measure of the accuracy of the Runge-Kutta order 4 DERIVE routine "RK4".
- Consider a linear system (although the routine works well with nonlinear systems):

$$\ddot{x} - 2\dot{x} - 3x = -4e^t$$
$$x(0) = 2, \quad \dot{x}(0) = 0.$$

- This has exact solution

$$x = e^{-t} + e^t = 2\cosh t.$$

Testing Accuracy of RK4

- Turn this IVP into a 2×2 system IVP as follows

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= 2y + 3x - 4e^t \\ x(0) = 2, &\quad y(0) = 0. \end{aligned}$$

- Differentiating the second equation yields the equivalent 3×3 system

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= z \\ \dot{z} &= 2z + 3y - 4e^t \\ x(0) = 2, \quad y(0) = 0, &\quad z(0) = 2. \end{aligned}$$

- The philosophy used here is that the numerical routine (usually Runge-Kutta (RK)) of any CAS working at a given precision returns two numerical solutions. How "close" these solutions are to one another can now be observed graphically and analytically.

Testing Accuracy of RK4

- Turn this IVP into a 2×2 system IVP as follows

$$\begin{aligned} \overset{\bullet}{x} &= y \\ \overset{\bullet}{y} &= 2y + 3x - 4e^t \\ x(0) = 2, & \quad y(0) = 0. \end{aligned}$$

- Differentiating the second equation yields the equivalent 3×3 system

$$\begin{aligned} \overset{\bullet}{x} &= y \\ \overset{\bullet}{y} &= z \\ \overset{\bullet}{z} &= 2z + 3y - 4e^t \\ x(0) = 2, \quad y(0) = 0, & \quad z(0) = 2. \end{aligned}$$

- The philosophy used here is that the numerical routine (usually Runge-Kutta (RK)) of any CAS working at a given precision returns two numerical solutions. How "close" these solutions are to one another can now be observed graphically and analytically.

Testing Accuracy of RK4

- Turn this IVP into a 2×2 system IVP as follows

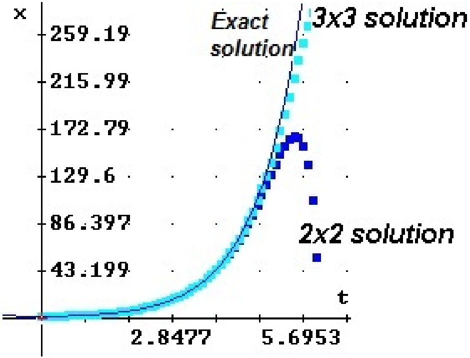
$$\begin{aligned} \overset{\bullet}{x} &= y \\ \overset{\bullet}{y} &= 2y + 3x - 4e^t \\ x(0) = 2, &\quad y(0) = 0. \end{aligned}$$

- Differentiating the second equation yields the equivalent 3×3 system

$$\begin{aligned} \overset{\bullet}{x} &= y \\ \overset{\bullet}{y} &= z \\ \overset{\bullet}{z} &= 2z + 3y - 4e^t \\ x(0) = 2, \quad y(0) = 0, &\quad z(0) = 2. \end{aligned}$$

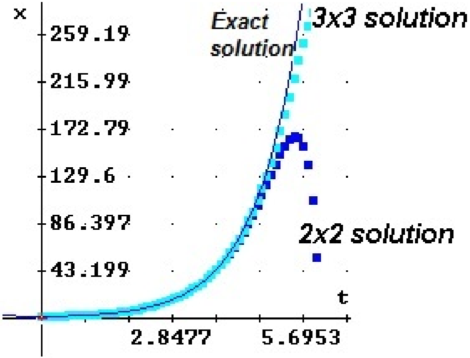
- The philosophy used here is that the numerical routine (usually Runge-Kutta (RK)) of any CAS working at a given precision returns two numerical solutions. How "close" these solutions are to one another can now be observed graphically and analytically.

Graphical results



- The accuracy of the routine can be gauged by observing the difference (**estimated error**) between the numerical solutions.

Graphical results



- The accuracy of the routine can be gauged by observing the difference (**estimated error**) between the numerical solutions.

Estimated error

- Because we know the exact solution we can tabulate the **actual error** and compare this with the **estimated error**.

- Hopefully the estimated error will tell us how accurate the routine will be with linear and nonlinear problems having no known exact solution.

Estimated error

- Because we know the exact solution we can tabulate the **actual error** and compare this with the **estimated error**.

t	Actual Error
1	$9.061932091 \cdot 10^{-5}$
1.1	0.0001261640890
1.2	0.0001745509993
1.3	0.0002403125765
1.4	0.0003295717135
1.5	0.0004505984321
1.6	0.0006145616139
1.7	0.0008365435693
1.8	0.001136909713
1.9	0.001543157466
2	0.002092412036

t	Estimated Error
1	$-5.943768169 \cdot 10^{-5}$
1.1	$-8.297205588 \cdot 10^{-5}$
1.2	−0.0001150529184
1.3	−0.0001587005486
1.4	−0.0002179952873
1.5	−0.0002984487011
1.6	−0.0004075046692
1.7	−0.0005552157854
1.8	−0.0007551564600
1.9	−0.001025655455
2	−0.001391459837

- Hopefully the estimated error will tell us how accurate the routine will be with linear and nonlinear problems having no known exact solution.

Estimated error

- Because we know the exact solution we can tabulate the **actual error** and compare this with the **estimated error**.

t	Actual Error
1	$9.061932091 \cdot 10^{-5}$
1.1	0.0001261640890
1.2	0.0001745509993
1.3	0.0002403125765
1.4	0.0003295717135
1.5	0.0004505984321
1.6	0.0006145616139
1.7	0.0008365435693
1.8	0.001136909713
1.9	0.001543157466
2	0.002092412036

t	Estimated Error
1	$-5.943768169 \cdot 10^{-5}$
1.1	$-8.297205588 \cdot 10^{-5}$
1.2	−0.0001150529184
1.3	−0.0001587005486
1.4	−0.0002179952873
1.5	−0.0002984487011
1.6	−0.0004075046692
1.7	−0.0005552157854
1.8	−0.0007551564600
1.9	−0.001025655455
2	−0.001391459837

- Hopefully the estimated error will tell us how accurate the routine will be with linear and nonlinear problems having no known exact solution.

Spin offs

- **Human resource development:**
  - **Professor Dr. Ansie Greeff**, my colleague and my doctoral student, has become well known in the field of mathematical biology. She has, with Temple Fay, developed a three competing species model with and without cropping.
  - Ansie co-supervised with Temple Fay the doctoral studies of our colleague from the University of Pretoria **Dr. Quay van der Hoff**. Quay has made numerous presentation at TIME conferences. Temple Fay and I supervised Quay during her masters studies at TUT\USM.

- In an article published in the **South African Journal of Science** Ansie and I discussed the method dealt with in my TIME2004 presentation and adapted it to work on a three-species model, demonstrating that the model is accurate to at least three significant figures. The model will have no problem predicting populations measured in thousands over a 20-year period in the Kruger National Park.

Spin offs

- **Human resource development:**
  - **Professor Dr. Ansie Greeff**, my colleague and my doctoral student, has become well known in the field of mathematical biology. She has, with Temple Fay, developed a three competing species model with and without cropping.
  - Ansie co-supervised with Temple Fay the doctoral studies of our colleague from the University of Pretoria **Dr. Quay van der Hoff**. Quay has made numerous presentation at TIME conferences. Temple Fay and I supervised Quay during her masters studies at TUT\USM.
- In an article published in the **South African Journal of Science** Ansie and I discussed the method dealt with in my TIME2004 presentation and adapted it to work on a three-species model, demonstrating that the model is accurate to at least three significant figures. The model will have no problem predicting populations measured in thousands over a 20-year period in the Kruger National Park.

Spin offs

- **Human resource development:**
  - **Professor Dr. Ansie Greeff**, my colleague and my doctoral student, has become well known in the field of mathematical biology. She has, with Temple Fay, developed a three competing species model with and without cropping.
  - Ansie co-supervised with Temple Fay the doctoral studies of our colleague from the University of Pretoria **Dr. Quay van der Hoff**. Quay has made numerous presentation at TIME conferences. Temple Fay and I supervised Quay during her masters studies at TUT\USM.
- In an article published in the **South African Journal of Science** Ansie and I discussed the method dealt with in my TIME2004 presentation and adapted it to work on a three-species model, demonstrating that the model is accurate to at least three significant figures. The model will have no problem predicting populations measured in thousands over a 20-year period in the Kruger National Park.

Spin offs

- **Human resource development:**
  - **Professor Dr. Ansie Greeff**, my colleague and my doctoral student, has become well known in the field of mathematical biology. She has, with Temple Fay, developed a three competing species model with and without cropping.
  - Ansie co-supervised with Temple Fay the doctoral studies of our colleague from the University of Pretoria **Dr. Quay van der Hoff**. Quay has made numerous presentation at TIME conferences. Temple Fay and I supervised Quay during her masters studies at TUT\USM.

- In an article published in the **South African Journal of Science** Ansie and I discussed the method dealt with in my TIME2004 presentation and adapted it to work on a three-species model, demonstrating that the model is accurate to at least three significant figures. The model will have no problem predicting populations measured in thousands over a 20-year period in the Kruger National Park.

Des-TIME, Dresden 2006: The mathematics and industrial applications associated with the singing wineglass using a CAS (with Michael Shatalov)

- **Equipment needed:**

- The qualitative effect of an earthquake or tornado moving down the edge of a bay or harbour can be observed in a classroom, with minimal apparatus.
- Indeed, anyone who causes a partially filled wineglass to "sing" (resonate) using a wet finger can observe the effect.
- If one carefully examines the surface of the liquid it is possible to see that four ridges or crests follow the finger around the rim. The wave patterns were already reported by Michael Faraday in 1831.

Des-TIME, Dresden 2006: The mathematics and industrial applications associated with the singing wineglass using a CAS (with Michael Shatalov)

- **Equipment needed:**
  1. Wineglass

- The qualitative effect of an earthquake or tornado moving down the edge of a bay or harbour can be observed in a classroom, with minimal apparatus.
- Indeed, anyone who causes a partially filled wineglass to "sing" (resonate) using a wet finger can observe the effect.
- If one carefully examines the surface of the liquid it is possible to see that four ridges or crests follow the finger around the rim. The wave patterns were already reported by Michael Faraday in 1831.

Des-TIME, Dresden 2006: The mathematics and industrial applications associated with the singing wineglass using a CAS (with Michael Shatalov)

- **Equipment needed:**
  1. Wineglass
  2. Milk or wine

- The qualitative effect of an earthquake or tornado moving down the edge of a bay or harbour can be observed in a classroom, with minimal apparatus.
- Indeed, anyone who causes a partially filled wineglass to "sing" (resonate) using a wet finger can observe the effect.
- If one carefully examines the surface of the liquid it is possible to see that four ridges or crests follow the finger around the rim. The wave patterns were already reported by Michael Faraday in 1831.

Des-TIME, Dresden 2006: The mathematics and industrial applications associated with the singing wineglass using a CAS (with Michael Shatalov)

- **Equipment needed:**
  1. Wineglass
  2. Milk or wine
  3. Turntable

- The qualitative effect of an earthquake or tornado moving down the edge of a bay or harbour can be observed in a classroom, with minimal apparatus.

- Indeed, anyone who causes a partially filled wineglass to "sing" (resonate) using a wet finger can observe the effect.

- If one carefully examines the surface of the liquid it is possible to see that four ridges or crests follow the finger around the rim. The wave patterns were already reported by Michael Faraday in 1831.

Des-TIME, Dresden 2006: The mathematics and industrial applications associated with the singing wineglass using a CAS (with Michael Shatalov)

- **Equipment needed:**
  1. Wineglass
  2. Milk or wine
  3. Turntable
  4. Camera

- The qualitative effect of an earthquake or tornado moving down the edge of a bay or harbour can be observed in a classroom, with minimal apparatus.

- Indeed, anyone who causes a partially filled wineglass to "sing" (resonate) using a wet finger can observe the effect.

- If one carefully examines the surface of the liquid it is possible to see that four ridges or crests follow the finger around the rim. The wave patterns were already reported by Michael Faraday in 1831.

Des-TIME, Dresden 2006: The mathematics and industrial applications associated with the singing wineglass using a CAS (with Michael Shatalov)

- **Equipment needed:**
  1. Wineglass
  2. Milk or wine
  3. Turntable
  4. Camera
  5. Any CAS
- The qualitative effect of an earthquake or tornado moving down the edge of a bay or harbour can be observed in a classroom, with minimal apparatus.
- Indeed, anyone who causes a partially filled wineglass to "sing" (resonate) using a wet finger can observe the effect.
- If one carefully examines the surface of the liquid it is possible to see that four ridges or crests follow the finger around the rim. The wave patterns were already reported by Michael Faraday in 1831.

Des-TIME, Dresden 2006: The mathematics and industrial applications associated with the singing wineglass using a CAS (with Michael Shatalov)

- **Equipment needed:**
  1. Wineglass
  2. Milk or wine
  3. Turntable
  4. Camera
  5. Any CAS
- The qualitative effect of an earthquake or tornado moving down the edge of a bay or harbour can be observed in a classroom, with minimal apparatus.
- Indeed, anyone who causes a partially filled wineglass to "sing" (resonate) using a wet finger can observe the effect.
- If one carefully examines the surface of the liquid it is possible to see that four ridges or crests follow the finger around the rim. The wave patterns were already reported by Michael Faraday in 1831.

Des-TIME, Dresden 2006: The mathematics and industrial applications associated with the singing wineglass using a CAS (with Michael Shatalov)

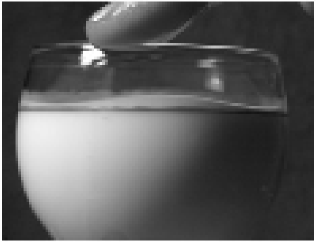
- **Equipment needed:**
  1. Wineglass
  2. Milk or wine
  3. Turntable
  4. Camera
  5. Any CAS
- The qualitative effect of an earthquake or tornado moving down the edge of a bay or harbour can be observed in a classroom, with minimal apparatus.
- Indeed, anyone who causes a partially filled wineglass to "sing" (resonate) using a wet finger can observe the effect.
- If one carefully examines the surface of the liquid it is possible to see that four ridges or crests follow the finger around the rim. The wave patterns were already reported by Michael Faraday in 1831.

Des-TIME, Dresden 2006: The mathematics and industrial applications associated with the singing wineglass using a CAS (with Michael Shatalov)

- **Equipment needed:**
  1. Wineglass
  2. Milk or wine
  3. Turntable
  4. Camera
  5. Any CAS
- The qualitative effect of an earthquake or tornado moving down the edge of a bay or harbour can be observed in a classroom, with minimal apparatus.
- Indeed, anyone who causes a partially filled wineglass to "sing" (resonate) using a wet finger can observe the effect.
- If one carefully examines the surface of the liquid it is possible to see that four ridges or crests follow the finger around the rim. The wave patterns were already reported by Michael Faraday in 1831.

Singing wineglass

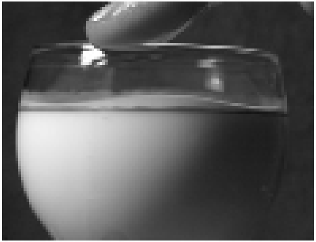
- Clearly the anti-node is below the energy source (the violin bow) when the glass is not rotating:

- The photograph on the left depicts a clockwise rotating glass and a static energy source (the finger).
- The Coriolis forces involved cause the anti-node to move to a fixed position away from the finger in the opposite direction to the rotation.
- In 1890 GH Bryan observed that he could not hear four beats when he turned a ringing wineglass through 360°.

Singing wineglass

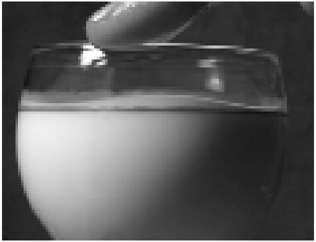
- Clearly the anti-node is below the energy source (the violin bow) when the glass is not rotating:



- The photograph on the left depicts a clockwise rotating glass and a static energy source (the finger).
- The Coriolis forces involved cause the anti-node to move to a fixed position away from the finger in the opposite direction to the rotation.
- In 1890 GH Bryan observed that he could not hear four beats when he turned a ringing wineglass through 360°.

Singing wineglass

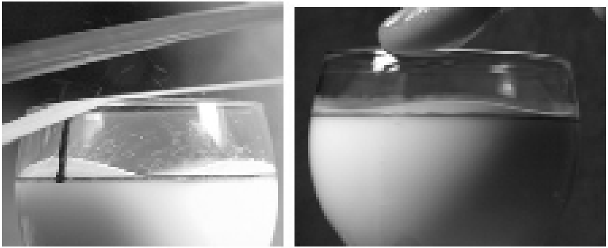
- Clearly the anti-node is below the energy source (the violin bow) when the glass is not rotating:



- The photograph on the left depicts a clockwise rotating glass and a static energy source (the finger).
- The Coriolis forces involved cause the anti-node to move to a fixed position away from the finger in the opposite direction to the rotation.
- In 1890 GH Bryan observed that he could not hear four beats when he turned a ringing wineglass through 360°.

Singing wineglass

- Clearly the anti-node is below the energy source (the violin bow) when the glass is not rotating:



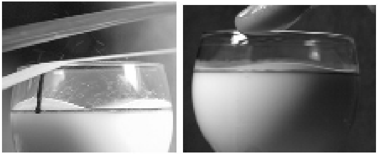
- The photograph on the left depicts a clockwise rotating glass and a static energy source (the finger).
- The Coriolis forces involved cause the anti-node to move to a fixed position away from the finger in the opposite direction to the rotation.
- In 1890 GH Bryan observed that he could not hear four beats when he turned a ringing wineglass through 360°.

Bryan's effect

- This occurs because of what is known today as **Bryan's Effect**: "When a vibrating body rotates, the rotation pattern rotates away from its position of rest at a rate **proportional** to the rotation rate of the body".
- Indeed, **Bryan's factor** is the constant

$$\eta = \frac{\text{Angular rate of the vibrating pattern}}{\text{Angular rate of the vibrating body}}$$

- **Are we seeing Bryan's Effect in the photo?** What is not clear from the photo is the fact that the anti-node moves away from below the finger to a fixed position.

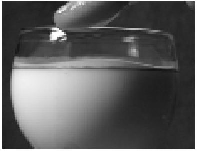


Bryan's effect

- This occurs because of what is known today as **Bryan's Effect**: "When a vibrating body rotates, the rotation pattern rotates away from its position of rest at a rate **proportional** to the rotation rate of the body".
- Indeed, **Bryan's factor** is the constant

$$\eta = \frac{\text{Angular rate of the vibrating pattern}}{\text{Angular rate of the vibrating body}}$$

- **Are we seeing Bryan's Effect in the photo?** What is not clear from the photo is the fact that the anti-node moves away from below the finger to a fixed position.

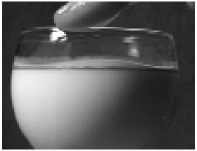


Bryan's effect

- This occurs because of what is known today as **Bryan's Effect**: "When a vibrating body rotates, the rotation pattern rotates away from its position of rest at a rate **proportional** to the rotation rate of the body".
- Indeed, **Bryan's factor** is the constant

$$\eta = \frac{\text{Angular rate of the vibrating pattern}}{\text{Angular rate of the vibrating body}}$$

- **Are we seeing Bryan's Effect in the photo?** What is not clear from the photo is the fact that the anti-node moves away from below the finger to a fixed position.

It turns out, after much investigation by our team, that this is Bryan's Effect modified by imperfections in the vibrating body such as mass and stiffness irregularities as well as anisotropic damping.

- For an ideal gyroscope without irregularities, if one knows the constant of proportionality (called **Bryan's Factor)** for a given substance, then one can use this effect for navigation.
  - Indeed the space shuttle and strato jets utilise vibratory gyroscopes based on Bryan's factor.

Navigation

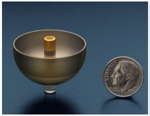
It turns out, after much investigation by our team, that this is Bryan's Effect modified by imperfections in the vibrating body such as mass and stiffness irregularities as well as anisotropic damping.

- For an ideal gyroscope without irregularities, if one knows the constant of proportionality (called **Bryan's Factor)** for a given substance, then one can use this effect for navigation.

    - Indeed the space shuttle and strato jets utilise vibratory gyroscopes based on Bryan's factor.

Navigation

It turns out, after much investigation by our team, that this is Bryan's Effect modified by imperfections in the vibrating body such as mass and stiffness irregularities as well as anisotropic damping.

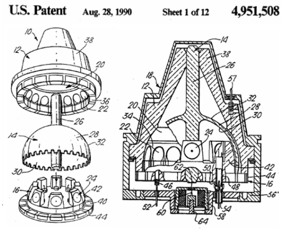
- For an ideal gyroscope without irregularities, if one knows the constant of proportionality (called **Bryan's Factor)** for a given substance, then one can use this effect for navigation.
  - Indeed the space shuttle and strato jets utilise vibratory gyroscopes based on Bryan's factor.
  -  

Bryan's effect dormant for 70 years

- Bryan's effect lay dormant for about 70 years before being uncovered in 1965. It took another 25 years to design and patent the first vibratory gyroscope in 1990.
- Spin-off publications

Bryan's effect dormant for 70 years

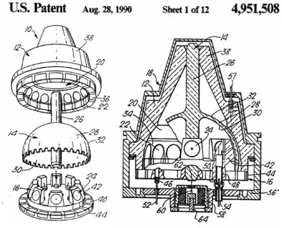
- Bryan's effect lay dormant for about 70 years before being uncovered in 1965. It took another 25 years to design and patent the first vibratory gyroscope in 1990.



- **Spin-off publications**

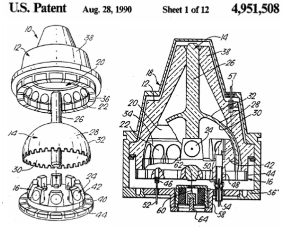
Bryan's effect dormant for 70 years

- Bryan's effect lay dormant for about 70 years before being uncovered in 1965. It took another 25 years to design and patent the first vibratory gyroscope in 1990.

- 

- **Spin-off publications**

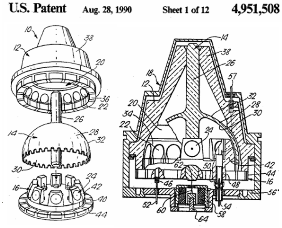
Bryan's effect dormant for 70 years

- Bryan's effect lay dormant for about 70 years before being uncovered in 1965. It took another 25 years to design and patent the first vibratory gyroscope in 1990.
- 

- **Spin-off publications**

1. **Am. J. Phys (AJP)** (2007). "A storm in a wineglass" with Temple Fay and Esme Voges

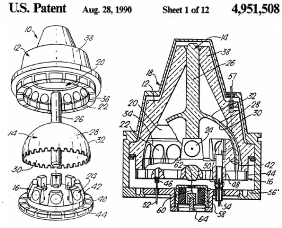
Bryan's effect dormant for 70 years

- Bryan's effect lay dormant for about 70 years before being uncovered in 1965. It took another 25 years to design and patent the first vibratory gyroscope in 1990.
- 

- **Spin-off publications**

1. **Am. J. Phys (AJP)** (2007). "A storm in a wineglass" with Temple Fay and Esme Voges
2. **IJMEST** (2008). "The singing wineglass: an exercise in mathematical modelling" with Esme Voges

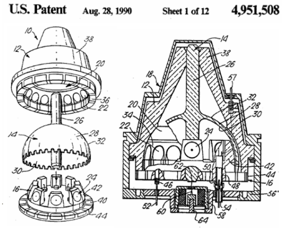
Bryan's effect dormant for 70 years

- Bryan's effect lay dormant for about 70 years before being uncovered in 1965. It took another 25 years to design and patent the first vibratory gyroscope in 1990.
- 

- **Spin-off publications**

1. **Am. J. Phys (AJP)** (2007). "A storm in a wineglass" with Temple Fay and Esme Voges
2. **IJMEST** (2008). "The singing wineglass: an exercise in mathematical modelling" with Esme Voges
3. **AJP** (2009). "Rotating structures and Bryan's effect" with Temple Fay and Michael Shatalov.

Bryan's effect dormant for 70 years

- Bryan's effect lay dormant for about 70 years before being uncovered in 1965. It took another 25 years to design and patent the first vibratory gyroscope in 1990.
- 

- **Spin-off publications**

1. **Am. J. Phys (AJP)** (2007). "A storm in a wineglass" with Temple Fay and Esme Voges
2. **IJMEST** (2008). "The singing wineglass: an exercise in mathematical modelling" with Esme Voges
3. **AJP** (2009). "Rotating structures and Bryan's effect" with Temple Fay and Michael Shatalov.
4. **Journal of Sound and Vibration** (2009). "Free vibration of rotating hollow spheres containing acoustic media" with Michael Shatalov, Charlotta Coetzee and Igor Fedotov.

Spin offs

1. The late Dr. Charlotta Coetzee, my colleague, my friend and my doctoral student, studied Bryan's effect in spherical bodies. She was co-supervised by Michael Shatalov.

Spin offs

1. The late Dr. Charlotta Coetzee, my colleague, my friend and my doctoral student, studied Bryan's effect in spherical bodies. She was co-supervised by Michael Shatalov.
2. Dr. Esme Voges my colleague and doctoral student, studied Bryan's effect in cylindrical and annular bodies. She was co-supervised by Michael Shatalov.

Spin offs

1. The late Dr. Charlotta Coetzee, my colleague, my friend and my doctoral student, studied Bryan's effect in spherical bodies. She was co-supervised by Michael Shatalov.
2. Dr. Esme Voges my colleague and doctoral student, studied Bryan's effect in cylindrical and annular bodies. She was co-supervised by Michael Shatalov.
3. **Buffelspoort TIME 2008, South Africa:**

Spin offs

1. The late Dr. Charlotta Coetzee, my colleague, my friend and my doctoral student, studied Bryan's effect in spherical bodies. She was co-supervised by Michael Shatalov.
2. Dr. Esme Voges my colleague and doctoral student, studied Bryan's effect in cylindrical and annular bodies. She was co-supervised by Michael Shatalov.
3. **Buffelspoort TIME 2008, South Africa:**
    1. Charlotta Coetzee and Hennie Kloppers assisted me with the organisation of the Buffelspoort-TIME 2008 conference.

Spin offs

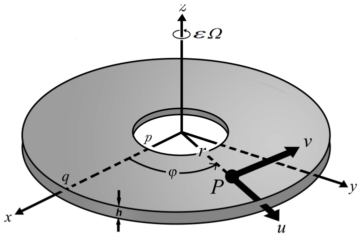
1. The late Dr. Charlotta Coetzee, my colleague, my friend and my doctoral student, studied Bryan's effect in spherical bodies. She was co-supervised by Michael Shatalov.
2. Dr. Esme Voges my colleague and doctoral student, studied Bryan's effect in cylindrical and annular bodies. She was co-supervised by Michael Shatalov.
3. **Buffelspoort TIME 2008, South Africa:**
    1. Charlotta Coetzee and Hennie Kloppers assisted me with the organisation of the Buffelspoort-TIME 2008 conference.
    2. I was co-author on a paper with the TIME stalwart Carl Leinbach.

Spin offs

1. The late Dr. Charlotta Coetzee, my colleague, my friend and my doctoral student, studied Bryan's effect in spherical bodies. She was co-supervised by Michael Shatalov.
2. Dr. Esme Voges my colleague and doctoral student, studied Bryan's effect in cylindrical and annular bodies. She was co-supervised by Michael Shatalov.
3. **Buffelspoort TIME 2008, South Africa:**
   1. Charlotta Coetzee and Hennie Kloppers assisted me with the organisation of the Buffelspoort-TIME 2008 conference.
   2. I was co-author on a paper with the TIME stalwart Carl Leinbach.
   3. I was co-author on ten other papers.

Malaga-TIME 2010, Spain: A CAS routine for obtaining eigenfunctions for Bryan's effect

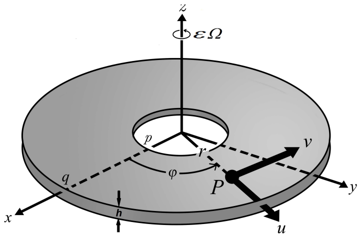
- Consider a vibrating, slowly rotating annular disc:



- The polar coordinates r and φ of the position of rest P of a vibrating particle in the annular disc of density ρ, height h, rotating slowly at rate $\varepsilon\Omega$. The vibrating particle at P has **radial** displacement u and **tangential** displacement v

Malaga-TIME 2010, Spain: A CAS routine for obtaining eigenfunctions for Bryan's effect

- Consider a vibrating, slowly rotating annular disc:



- The polar coordinates r and φ of the position of rest P of a vibrating particle in the annular disc of density ρ, height h, rotating slowly at rate $\varepsilon\Omega$. The vibrating particle at P has **radial** displacement u and **tangential** displacement v

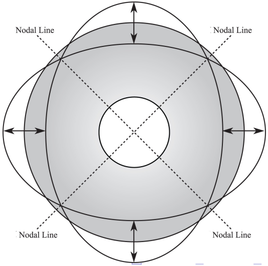
Eigenfunctions

- If $U(r)$ and $V(r)$ the eigenfunctions associated with the eigenvalue of vibration ω^2, assume that

$$u = U(r)[C(t)\cos m\varphi + S(t)\sin m\varphi]$$
$$v = V(r)[C(t)\sin m\varphi - S(t)\cos m\varphi]$$

where, $C(t)$ and $S(t)$ are to be determined and m is the circumferential wave number.

- It has been demonstrated that $\omega^2 = \frac{2\int_p^q UVrdr}{\int_p^q (U^2+V^2)rdr} = \frac{I_2}{I_0}$.
  - When $m = 2$ the vibration pattern has four nodes:



Eigenfunctions

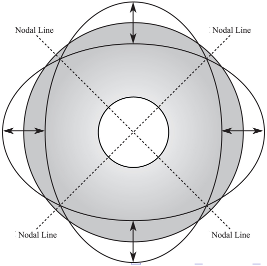
- If $U(r)$ and $V(r)$ the eigenfunctions associated with the eigenvalue of vibration ω^2, assume that

$$u = U(r)[C(t)\cos m\varphi + S(t)\sin m\varphi]$$
$$v = V(r)[C(t)\sin m\varphi - S(t)\cos m\varphi]$$

where, $C(t)$ and $S(t)$ are to be determined and m is the circumferential wave number.

- It has been demonstrated that $\omega^2 = \dfrac{2\int_p^q UVrdr}{\int_p^q (U^2+V^2)rdr} = \dfrac{I_2}{I_0}$.

  - When $m = 2$ the vibration pattern has four nodes:



Eigenfunctions

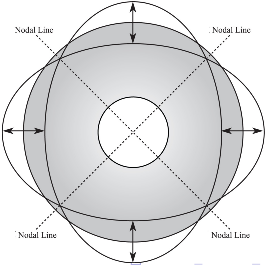
- If $U(r)$ and $V(r)$ the eigenfunctions associated with the eigenvalue of vibration ω^2, assume that

$$u = U(r)[C(t)\cos m\varphi + S(t)\sin m\varphi]$$
$$v = V(r)[C(t)\sin m\varphi - S(t)\cos m\varphi]$$

where, $C(t)$ and $S(t)$ are to be determined and m is the circumferential wave number.

- It has been demonstrated that $\omega^2 = \dfrac{2\int_p^q UVrdr}{\int_p^q (U^2+V^2)rdr} = \dfrac{I_2}{I_0}$.

  - When $m = 2$ the vibration pattern has four nodes:



Eigenfunctions(continued)

- Applying Newton's laws of motion to the mass element at point P, one arrives at two coupled PDE, and, assuming harmonic solutions, we arrive at a system of two coupled *linear, ordinary differential equations* (ODEs)

$$2r^2U'' + 2rU' + \left(a\omega^2 r^2 - b\right)U + crV' - dV = 0;$$
$$(1-\mu)r^2V'' + (1-\mu)rV' + \left(a\omega^2 r^2 - e\right)V - crU' - dU = 0;$$

- where, with With ρ mass density, E Young's modulus and μ Poisson's ratio we have

$$a = \frac{2\left(1-\mu^2\right)\rho}{E};\, b = m^2(1-\mu) + 2;$$
$$c = m(1+\mu);\, d = m(3-\mu);$$
$$e = 2m^2 + 1 - \mu.$$

Eigenfunctions(continued)

- Applying Newton's laws of motion to the mass element at point P, one arrives at two coupled PDE, and, assuming harmonic solutions, we arrive at a system of two coupled *linear, ordinary differential equations* (ODEs)

$$2r^2U'' + 2rU' + \left(a\omega^2 r^2 - b\right) U + crV' - dV = 0;$$
$$(1-\mu)r^2V'' + (1-\mu)rV' + \left(a\omega^2 r^2 - e\right) V - crU' - dU = 0;$$

- where, with With ρ mass density, E Young's modulus and μ Poisson's ratio we have

$$a = \frac{2\left(1-\mu^2\right)\rho}{E}; b = m^2(1-\mu) + 2;$$
$$c = m(1+\mu); d = m(3-\mu);$$
$$e = 2m^2 + 1 - \mu.$$

Iteration process

- For a given substance and circumferential wave number m, the constants a, b, c, d, e are known. If a value is assigned to ω, then the NDSolve routine of Mathematica will produce a numerical solution for suitable boundary conditions.

- For a **guess value** of ω, MATHEMATICA® stores U and V as **interpolating functions**. A new value for ω may now be generated by MATHEMATICA® via the known formula

$$\omega = \sqrt{\frac{2\int_p^q UVrdr}{\int_p^q \left(U^2 + V^2\right) rdr}}.$$

- An iteration process is now carried out that determines a sequence of values for ω that hopefully converges to a satisfactory number of digits.

- We obtained good approximate values for the guess eigenvalues using the Rayleigh-Ritz method.

Iteration process

- For a given substance and circumferential wave number m, the constants a, b, c, d, e are known. If a value is assigned to ω, then the NDSolve routine of Mathematica will produce a numerical solution for suitable boundary conditions.
- For a **guess value** of ω, MATHEMATICA® stores U and V as **interpolating functions**. A new value for ω may now be generated by MATHEMATICA® via the known formula

$$\omega = \sqrt{\frac{2 \int_p^q UVrdr}{\int_p^q (U^2 + V^2)\, rdr}}.$$

- An iteration process is now carried out that determines a sequence of values for ω that hopefully converges to a satisfactory number of digits.
- We obtained good approximate values for the guess eigenvalues using the Rayleigh-Ritz method.

Iteration process

- For a given substance and circumferential wave number m, the constants a, b, c, d, e are known. If a value is assigned to ω, then the NDSolve routine of Mathematica will produce a numerical solution for suitable boundary conditions.

- For a **guess value** of ω, MATHEMATICA® stores U and V as **interpolating functions**. A new value for ω may now be generated by MATHEMATICA® via the known formula

$$\omega = \sqrt{\frac{2\int_p^q UVrdr}{\int_p^q \left(U^2 + V^2\right) rdr}}.$$

- An iteration process is now carried out that determines a sequence of values for ω that hopefully converges to a satisfactory number of digits.

- We obtained good approximate values for the guess eigenvalues using the Rayleigh-Ritz method.

Iteration process

- For a given substance and circumferential wave number m, the constants a, b, c, d, e are known. If a value is assigned to ω, then the NDSolve routine of Mathematica will produce a numerical solution for suitable boundary conditions.
- For a **guess value** of ω, $\text{MATHEMATICA}_{®}$ stores U and V as **interpolating functions**. A new value for ω may now be generated by $\text{MATHEMATICA}_{®}$ via the known formula

$$\omega = \sqrt{\frac{2 \int_p^q UVrdr}{\int_p^q \left(U^2 + V^2\right) rdr}}.$$

- An iteration process is now carried out that determines a sequence of values for ω that hopefully converges to a satisfactory number of digits.
- We obtained good approximate values for the guess eigenvalues using the Rayleigh-Ritz method.

Summary of the iteration process

- With constants a, b, c, d determined, determine a good guess value ω_0 and solve

$$2r^2U'' + 2rU' + \left(a\omega_0^2 r^2 - b\right) U + crV' - dV = 0;$$

$$(1-\mu)r^2V'' + (1-\mu)rV' + \left(a\omega_0^2 r^2 - e\right) V - crU' - dU = 0;$$

obtaining solutions U_1 and V_1 say

- A new value for ω may now be generated by MATHEMATICA, say ω_1

$$\omega_1 = \sqrt{\frac{2\int_p^q U_1 V_1 r dr}{\int_p^q \left(U_1^2 + V_1^2\right) r dr}}.$$

- One obtains a sequence of angular frequencies $\omega_0, \omega_1, \omega_2 \cdots$ that hopefully converges to a satisfactory amount of significant figures.

Summary of the iteration process

- With constants a, b, c, d determined, determine a good guess value ω_0 and solve

$$2r^2U'' + 2rU' + \left(a\omega_0^2 r^2 - b\right)U + crV' - dV = 0;$$

$$(1-\mu)r^2V'' + (1-\mu)rV' + \left(a\omega_0^2 r^2 - e\right)V - crU' - dU = 0;$$

obtaining solutions U_1 and V_1 say

- A new value for ω may now be generated by MATHEMATICA, say ω_1

$$\omega_1 = \sqrt{\frac{2\int_p^q U_1 V_1 \, rdr}{\int_p^q \left(U_1^2 + V_1^2\right) rdr}}.$$

- One obtains a sequence of angular frequencies $\omega_0, \omega_1, \omega_2 \cdots$ that hopefully converges to a satisfactory amount of significant figures.

Summary of the iteration process

- With constants a, b, c, d determined, determine a good guess value ω_0 and solve

$$2r^2U'' + 2rU' + \left(a\omega_0^2 r^2 - b\right) U + crV' - dV = 0;$$

$$(1-\mu)r^2V'' + (1-\mu)rV' + \left(a\omega_0^2 r^2 - e\right) V - crU' - dU = 0;$$

obtaining solutions U_1 and V_1 say

- A new value for ω may now be generated by MATHEMATICA, say ω_1

$$\omega_1 = \sqrt{\frac{2\int_p^q U_1 V_1 r dr}{\int_p^q \left(U_1^2 + V_1^2\right) r dr}}.$$

- One obtains a sequence of angular frequencies $\omega_0, \omega_1, \omega_2 \cdots$ that hopefully converges to a satisfactory amount of significant figures.

Spin-off publications

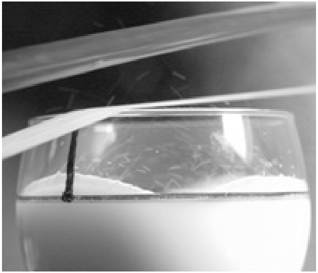
- Article published in 2013 in the **Journal of Sound and Vibration**: "Bryan's effect and isotropic nonlinear damping" with Michael Shatalov and Alexander Manzhirov.
- Article published in 2014 in the **Journal of Applied Mathematics and Computation**: "On numerically solving an eigenvalue problem arising in a resonator gyroscope" with Michael Shatalov and Temple Fay

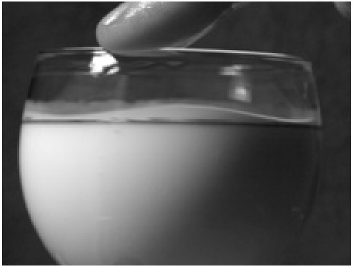
Spin-off publications

- Article published in 2013 in the **Journal of Sound and Vibration**: "Bryan's effect and isotropic nonlinear damping" with Michael Shatalov and Alexander Manzhirov.
- Article published in 2014 in the **Journal of Applied Mathematics and Computation**: "On numerically solving an eigenvalue problem arising in a resonator gyroscope" with Michael Shatalov and Temple Fay

TIME 2012 Tartu, Estonia: Using Fourier series to analyse mass imperfections in vibratory gyroscopes

- At this conference we considered once more the singing wineglass:

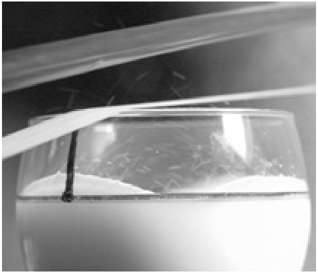


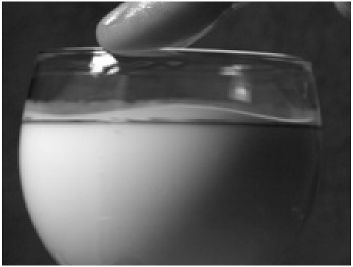


- Theoretically in an ideal wineglass we should be observing **Bryan's Effect** and the node should move away at a constant rotation rate, not be **captured** in a fixed position.
- By introducing density variations using a **Fourier series**, we are able to conduct a numerical experiment that predicts a **"capture effect"**.

TIME 2012 Tartu, Estonia: Using Fourier series to analyse mass imperfections in vibratory gyroscopes

- At this conference we considered once more the singing wineglass:





- Theoretically in an ideal wineglass we should be observing **Bryan's Effect** and the node should move away at a constant rotation rate, not be **captured** in a fixed position.
- By introducing density variations using a **Fourier series**, we are able to conduct a numerical experiment that predicts a **"capture effect"**.

TIME 2012 Tartu, Estonia: Using Fourier series to analyse mass imperfections in vibratory gyroscopes

- At this conference we considered once more the singing wineglass:

- Theoretically in an ideal wineglass we should be observing **Bryan's Effect** and the node should move away at a constant rotation rate, not be **captured** in a fixed position.
- By introducing density variations using a **Fourier series**, we are able to conduct a numerical experiment that predicts a **"capture effect"**.

- Recall that we set

$$u = U(r)[C(t)\cos m\varphi + S(t)\sin m\varphi]$$
$$v = V(r)[C(t)\sin m\varphi - S(t)\cos m\varphi]$$

- Notice that, for instance

$$\dot{v}^2 = \frac{1}{2}V^2[\dot{C}^2(1-\cos 2m\varphi) + \dot{S}^2(1+\cos 2m\varphi) - 2\dot{C}\dot{S}\sin 2m\varphi]$$

- The kinetic energy of all of the particles in the disc is given by

$$E_k \approx \frac{h}{2}\int_0^{2\pi}\int_p^q \rho(\varphi)\left[\left(\dot{u}^2+\dot{v}^2\right) + 2\varepsilon\Omega\left(u\dot{v}-\dot{u}v\right) + 2\varepsilon\Omega\dot{v}r\right] rdrd\varphi.$$

- Hence we should be able to use orthogonality to simplify matters.

- Recall that we set

$$u = U(r)[C(t)\cos m\varphi + S(t)\sin m\varphi]$$
$$v = V(r)[C(t)\sin m\varphi - S(t)\cos m\varphi]$$

- Notice that, for instance

$$\dot{v}^2 = \frac{1}{2}V^2[\dot{C}^2(1-\cos 2m\varphi) + \dot{S}^2(1+\cos 2m\varphi) - 2\dot{C}\dot{S}\sin 2m\varphi]$$

- The kinetic energy of all of the particles in the disc is given by

$$E_k \approx \frac{h}{2}\int_0^{2\pi}\int_p^q \rho(\varphi)\left[\left(\dot{u}^2+\dot{v}^2\right) + 2\varepsilon\Omega\left(u\dot{v} - \dot{u}v\right) + 2\varepsilon\Omega\dot{v}r\right] r dr d\varphi.$$

- Hence we should be able to use orthogonality to simplify matters.

- Recall that we set

$$u = U(r)[C(t)\cos m\varphi + S(t)\sin m\varphi]$$
$$v = V(r)[C(t)\sin m\varphi - S(t)\cos m\varphi]$$

- Notice that, for instance

$$\dot{v}^2 = \frac{1}{2}V^2[\dot{C}^2(1-\cos 2m\varphi) + \dot{S}^2(1+\cos 2m\varphi) - 2\dot{C}\dot{S}\sin 2m\varphi]$$

- The kinetic energy of all of the particles in the disc is given by

$$E_k \approx \frac{h}{2}\int_0^{2\pi}\int_p^q \rho(\varphi)\left[\left(\dot{u}^2+\dot{v}^2\right) + 2\varepsilon\Omega\left(u\dot{v}-\dot{u}v\right) + 2\varepsilon\Omega\dot{v}r\right] r dr d\varphi.$$

- Hence we should be able to use orthogonality to simplify matters.

- Recall that we set

$$u = U(r)[C(t)\cos m\varphi + S(t)\sin m\varphi]$$
$$v = V(r)[C(t)\sin m\varphi - S(t)\cos m\varphi]$$

- Notice that, for instance

$$\dot{v}^2 = \frac{1}{2}V^2[\dot{C}^2(1-\cos 2m\varphi) + \dot{S}^2(1+\cos 2m\varphi) - 2\dot{C}\dot{S}\sin 2m\varphi]$$

- The kinetic energy of all of the particles in the disc is given by

$$E_k \approx \frac{h}{2}\int_0^{2\pi}\int_p^q \rho(\varphi)\left[\left(\dot{u}^2+\dot{v}^2\right) + 2\varepsilon\Omega\left(u\dot{v}-\dot{u}v\right) + 2\varepsilon\Omega\dot{v}r\right] rdrd\varphi.$$

- Hence we should be able to use orthogonality to simplify matters.

Fourier series

- If we consider the density as a Fourier Series

$$\rho(\varphi) = \rho_0(1 + \varepsilon \sum_{k=1}^{\infty}(a_k \cos k\varphi + b_k \sin k\varphi))$$

- Then only the zeroth and $2m^{th}$ harmonics play a roll in the kinetic energy expression and so we might as well choose

$$\rho(\varphi) := \rho_0 \left[1 + \varepsilon \frac{4I_0}{I_3}\left(\rho_c \cos 2m\varphi + \rho_s \sin 2m\varphi\right)\right].$$

- where for simplification purposes, we have set $a_{2m} = 4\frac{I_0}{I_3}\rho_c$ and $b_{2m} = 4\frac{I_0}{I_3}\rho_s$ with ρ_c and ρ_c dimensionless constants that we will call the coefficients of the $2m^{th}$ harmonics. The ratio $\frac{I_0}{I_3}$ is a ratio of definite integrals (we have already seen something similar with $\frac{I_2}{I_0} = \omega^2 = \frac{2\int_p^q UVrdr}{\int_p^q (U^2+V^2)rdr}$).

Fourier series

- If we consider the density as a Fourier Series

$$\rho(\varphi) = \rho_0(1 + \varepsilon \sum_{k=1}^{\infty}(a_k \cos k\varphi + b_k \sin k\varphi))$$

- Then only the zeroth and $2m^{th}$ harmonics play a roll in the kinetic energy expression and so we might as well choose

$$\rho(\varphi) := \rho_0 \left[1 + \varepsilon \frac{4 I_0}{I_3} \left(\rho_c \cos 2m\varphi + \rho_s \sin 2m\varphi\right)\right].$$

- where for simplification purposes, we have set $a_{2m} = 4\frac{I_0}{I_3}\rho_c$ and $b_{2m} = 4\frac{I_0}{I_3}\rho_s$ with ρ_c and ρ_c dimensionless constants that we will call the coefficients of the $2m^{th}$ harmonics. The ratio $\frac{I_0}{I_3}$ is a ratio of definite integrals (we have already seen something similar with $\frac{I_2}{I_0} = \omega^2 = \frac{2\int_p^q UVrdr}{\int_p^q (U^2+V^2)rdr}$).

Fourier series

- If we consider the density as a Fourier Series

$$\rho(\varphi) = \rho_0(1 + \varepsilon \sum_{k=1}^{\infty}(a_k \cos k\varphi + b_k \sin k\varphi))$$

- Then only the zeroth and $2m^{th}$ harmonics play a roll in the kinetic energy expression and so we might as well choose

$$\rho(\varphi) := \rho_0 \left[1 + \varepsilon \frac{4I_0}{I_3}\left(\rho_c \cos 2m\varphi + \rho_s \sin 2m\varphi\right)\right].$$

- where for simplification purposes, we have set $a_{2m} = 4\frac{I_0}{I_3}\rho_c$ and $b_{2m} = 4\frac{I_0}{I_3}\rho_s$ with ρ_c and ρ_c dimensionless constants that we will call the coefficients of the $2m^{th}$ harmonics. The ratio $\frac{I_0}{I_3}$ is a ratio of definite integrals (we have already seen something similar with $\frac{I_2}{I_0} = \omega^2 = \frac{2\int_p^q UVrdr}{\int_p^q (U^2+V^2)rdr}$).

Kinetic energy

- With an eye on the Lagrange-Euler equations, we used MATHEMATICA® to calculate the various parts of the kinetic energy. For instance:

In[128]:= FullSimplify[

$$\frac{\rho_0\, h}{2}\left(4\, \epsilon\, \frac{I_0}{I_3}\, \rho_c\right) \int_0^{2\pi}\int_p^q \mathrm{Cos}[2\, m\, \varphi]\left((\partial_t u[r,\varphi,t])^2 + (\partial_t v[r,\varphi,t])^2\right) r\, dr\, d\varphi\ ,\quad m \in \mathrm{Integers}\Big]$$

Out[128]= $\left\{\frac{1}{I_3} 2\, h\, \epsilon \left(\int_p^q \frac{1}{2}\pi\, r\, \left(U[r]^2 - V[r]^2\right)\left(C'[t]^2 - S'[t]^2\right) dr\right) I_0\, \rho_0\, \rho_c\right\}$

-

Kinetic energy

- With an eye on the Lagrange-Euler equations, we used MATHEMATICA® to calculate the various parts of the kinetic energy. For instance:

In[128]:= FullSimplify[

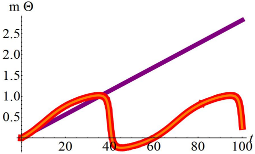
$$\frac{\rho_0\, h}{2}\left(4 \in \frac{I_0}{I_3}\rho_c\right)\int_0^{2\pi}\int_p^q \mathrm{Cos}[2\, m\, \varphi]\left((\partial_t u[r,\varphi,t])^2 + (\partial_t v[r,\varphi,t])^2\right) r\, dr\, d\varphi\,,\quad m \in \mathrm{Integers}]$$

Out[128]= $\left\{\frac{1}{I_3} 2\, h \in \left(\int_p^q \frac{1}{2}\pi\, r\, \left(U[r]^2 - V[r]^2\right)\left(C'[t]^2 - S'[t]^2\right) dr\right) I_0\, \rho_0\, \rho_c\right\}$

-

Capture effect

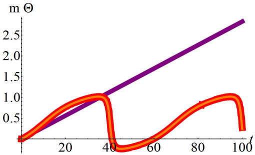
- Considering the vibration pattern for the m^{th} circumferential wave number $m\Theta$, we have chosen ODE coefficients that show the following **capture effect**:

- 

- The purple line indicates the ideal case with the precession angle $m\Theta$ for a slowly rotating disk changing linearly while the precession angle for a slowly rotating disk with light mass imperfections is "captured" periodically (the red/orange curve).
- **Spin-off publication**
  - Article published in 2014 in the **Journal of Symbolic Computation (TIME 2012 Proceedings)**: "Using Fourier series to analyse mass imperfections in vibratory gyroscopes" with Michael Shatalov and Charlotta Coetzee.

Capture effect

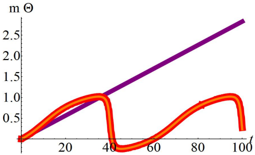
- Considering the vibration pattern for the m^{th} circumferential wave number $m\Theta$, we have chosen ODE coefficients that show the following **capture effect**:



- The purple line indicates the ideal case with the precession angle $m\Theta$ for a slowly rotating disk changing linearly while the precession angle for a slowly rotating disk with light mass imperfections is "captured" periodically (the red/orange curve).
- **Spin-off publication**
  - Article published in 2014 in the **Journal of Symbolic Computation (TIME 2012 Proceedings)**: "Using Fourier series to analyse mass imperfections in vibratory gyroscopes" with Michael Shatalov and Charlotta Coetzee.

Capture effect

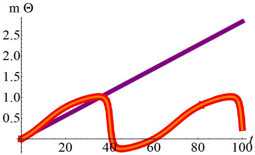
- Considering the vibration pattern for the m^{th} circumferential wave number $m\Theta$, we have chosen ODE coefficients that show the following **capture effect**:

- 

- The purple line indicates the ideal case with the precession angle $m\Theta$ for a slowly rotating disk changing linearly while the precession angle for a slowly rotating disk with light mass imperfections is "captured" periodically (the red/orange curve).
- **Spin-off publication**
  - Article published in 2014 in the **Journal of Symbolic Computation (TIME 2012 Proceedings)**: "Using Fourier series to analyse mass imperfections in vibratory gyroscopes" with Michael Shatalov and Charlotta Coetzee.

Capture effect

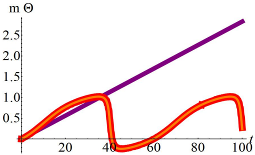
- Considering the vibration pattern for the m^{th} circumferential wave number $m\Theta$, we have chosen ODE coefficients that show the following **capture effect**:



- The purple line indicates the ideal case with the precession angle $m\Theta$ for a slowly rotating disk changing linearly while the precession angle for a slowly rotating disk with light mass imperfections is "captured" periodically (the red/orange curve).
- **Spin-off publication**
  - Article published in 2014 in the **Journal of Symbolic Computation (TIME 2012 Proceedings)**: "Using Fourier series to analyse mass imperfections in vibratory gyroscopes" with Michael Shatalov and Charlotta Coetzee.

Capture effect

- Considering the vibration pattern for the m^{th} circumferential wave number $m\Theta$, we have chosen ODE coefficients that show the following **capture effect**:

- 

- The purple line indicates the ideal case with the precession angle $m\Theta$ for a slowly rotating disk changing linearly while the precession angle for a slowly rotating disk with light mass imperfections is "captured" periodically (the red/orange curve).
- **Spin-off publication**
  - Article published in 2014 in the **Journal of Symbolic Computation (TIME 2012 Proceedings)**: "Using Fourier series to analyse mass imperfections in vibratory gyroscopes" with Michael Shatalov and Charlotta Coetzee.

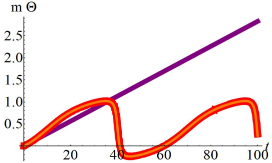
TIME 2014, Krems, Austria: Controlling mass-imperfections in vibratory gyroscopes

- If a disc gyroscope, with known Bryan's factor η, is mounted in a spacecraft and the vibration pattern of the gyroscope is observed, then a slow rate of rotation rate of the craft $\varepsilon\Omega$ may be measured via

$$\varepsilon\Omega = \frac{\text{Rate of rotation of the vibrating pattern of the gyroscope}}{\eta}.$$

-

Recall the graph of the "capture effect".caused by mass (and possibly other) imperfections.

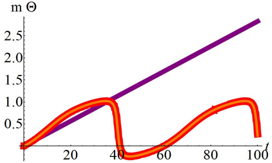


TIME 2014, Krems, Austria: Controlling mass-imperfections in vibratory gyroscopes

- If a disc gyroscope, with known Bryan's factor η, is mounted in a spacecraft and the vibration pattern of the gyroscope is observed, then a slow rate of rotation rate of the craft $\varepsilon\Omega$ may be measured via

$$\varepsilon\Omega = \frac{\text{Rate of rotation of the vibrating pattern of the gyroscope}}{\eta}.$$

- Recall the graph of the "capture effect".caused by mass (and possibly other) imperfections.



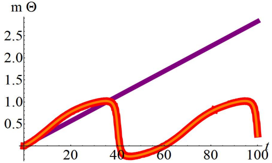
TIME 2014, Krems, Austria: Controlling mass-imperfections in vibratory gyroscopes

- If a disc gyroscope, with known Bryan's factor η, is mounted in a spacecraft and the vibration pattern of the gyroscope is observed, then a slow rate of rotation rate of the craft $\varepsilon\Omega$ may be measured via

$$\varepsilon\Omega = \frac{\text{Rate of rotation of the vibrating pattern of the gyroscope}}{\eta}.$$

-

Recall the graph of the "capture effect".caused by mass (and possibly other) imperfections.



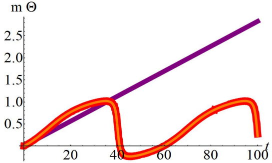
TIME 2014, Krems, Austria: Controlling mass-imperfections in vibratory gyroscopes

- If a disc gyroscope, with known Bryan's factor η, is mounted in a spacecraft and the vibration pattern of the gyroscope is observed, then a slow rate of rotation rate of the craft $\varepsilon\Omega$ may be measured via

$$\varepsilon\Omega = \frac{\text{Rate of rotation of the vibrating pattern of the gyroscope}}{\eta}.$$

-

Recall the graph of the "capture effect".caused by mass (and possibly other) imperfections.

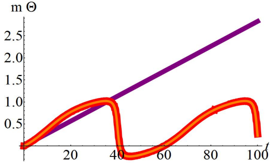


TIME 2014, Krems, Austria: Controlling mass-imperfections in vibratory gyroscopes

- If a disc gyroscope, with known Bryan's factor η, is mounted in a spacecraft and the vibration pattern of the gyroscope is observed, then a slow rate of rotation rate of the craft $\varepsilon\Omega$ may be measured via

$$\varepsilon\Omega = \frac{\text{Rate of rotation of the vibrating pattern of the gyroscope}}{\eta}.$$

- Recall the graph of the "capture effect".caused by mass (and possibly other) imperfections.



Frequency split

- Mass imperfections cause **beats** (that can be heard in large VGs). Indeed, we have shown that the frequency of the beats is given by

$$f = \frac{\varepsilon\omega\sqrt{\rho_c^2 + \rho_s^2}}{2\pi}$$

where the coefficients of the $2m^{th}$ harmonics come from the Fourier series for the density

$$\rho(\varphi) := \rho_0 \left[1 + \varepsilon \frac{4 I_0}{I_3} \left(\rho_c \cos 2m\varphi + \rho_s \sin 2m\varphi\right)\right].$$

- This frequency splitting causes the vibratory gyroscope to deviate from ideal behaviour where Bryan's factor can be used for navigation purposes.

Frequency split

- Mass imperfections cause **beats** (that can be heard in large VGs). Indeed, we have shown that the frequency of the beats is given by

$$f = \frac{\varepsilon\omega\sqrt{\rho_c^2 + \rho_s^2}}{2\pi}$$

where the coefficients of the $2m^{th}$ harmonics come from the Fourier series for the density

$$\rho(\varphi) := \rho_0 \left[1 + \varepsilon\frac{4I_0}{I_3}\left(\rho_c \cos 2m\varphi + \rho_s \sin 2m\varphi\right)\right].$$

- This frequency splitting causes the vibratory gyroscope to deviate from ideal behaviour where Bryan's factor can be used for navigation purposes.

Equations of motion

- Indeed, the deviation from ideal behaviour can be observed from the equations of motion.

- Indeed, those of an ideal annular VG are

$$\begin{pmatrix} \ddot{C} \\ \ddot{S} \end{pmatrix} + \omega^2 \begin{pmatrix} C \\ S \end{pmatrix} = 2\eta\varepsilon\Omega \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \dot{C} \\ \dot{S} \end{pmatrix},$$

- while those of a mass-imperfect annular VG are

$$\begin{pmatrix} \ddot{C} \\ \ddot{S} \end{pmatrix} + \omega^2 \begin{pmatrix} 1-\varepsilon\rho_c & -\varepsilon\rho_s \\ -\varepsilon\rho_s & \varepsilon\rho_c \end{pmatrix} \begin{pmatrix} C \\ S \end{pmatrix} = 2\eta\varepsilon\Omega \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \dot{C} \\ \dot{S} \end{pmatrix}.$$

- We proceed to eliminate the frequency split.

Equations of motion

- Indeed, the deviation from ideal behaviour can be observed from the equations of motion.
- Indeed, those of an ideal annular VG are

$$\begin{pmatrix} \ddot{C} \\ \ddot{S} \end{pmatrix} + \omega^2 \begin{pmatrix} C \\ S \end{pmatrix} = 2\eta\varepsilon\Omega \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \dot{C} \\ \dot{S} \end{pmatrix},$$

- while those of a mass-imperfect annular VG are

$$\begin{pmatrix} \ddot{C} \\ \ddot{S} \end{pmatrix} + \omega^2 \begin{pmatrix} 1-\varepsilon\rho_c & -\varepsilon\rho_s \\ -\varepsilon\rho_s & \varepsilon\rho_c \end{pmatrix} \begin{pmatrix} C \\ S \end{pmatrix} = 2\eta\varepsilon\Omega \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \dot{C} \\ \dot{S} \end{pmatrix}.$$

- We proceed to eliminate the frequency split.

Equations of motion

- Indeed, the deviation from ideal behaviour can be observed from the equations of motion.

- Indeed, those of an ideal annular VG are

$$\begin{pmatrix} \ddot{C} \\ \ddot{S} \end{pmatrix} + \omega^2 \begin{pmatrix} C \\ S \end{pmatrix} = 2\eta\varepsilon\Omega \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \dot{C} \\ \dot{S} \end{pmatrix},$$

- while those of a mass-imperfect annular VG are

$$\begin{pmatrix} \ddot{C} \\ \ddot{S} \end{pmatrix} + \omega^2 \begin{pmatrix} 1 - \varepsilon\rho_c & -\varepsilon\rho_s \\ -\varepsilon\rho_s & \varepsilon\rho_c \end{pmatrix} \begin{pmatrix} C \\ S \end{pmatrix} = 2\eta\varepsilon\Omega \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \dot{C} \\ \dot{S} \end{pmatrix}.$$

- We proceed to eliminate the frequency split.

Equations of motion

- Indeed, the deviation from ideal behaviour can be observed from the equations of motion.

- Indeed, those of an ideal annular VG are

$$\begin{pmatrix} \ddot{C} \\ \ddot{S} \end{pmatrix} + \omega^2 \begin{pmatrix} C \\ S \end{pmatrix} = 2\eta\varepsilon\Omega \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \dot{C} \\ \dot{S} \end{pmatrix},$$

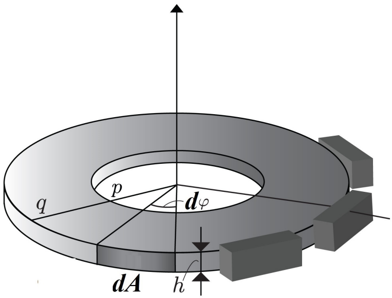
- while those of a mass-imperfect annular VG are

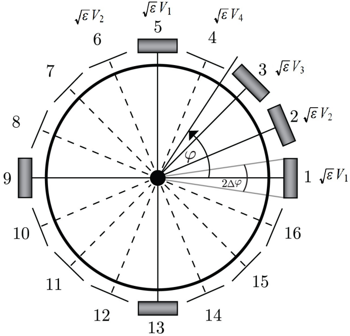
$$\begin{pmatrix} \ddot{C} \\ \ddot{S} \end{pmatrix} + \omega^2 \begin{pmatrix} 1 - \varepsilon\rho_c & -\varepsilon\rho_s \\ -\varepsilon\rho_s & \varepsilon\rho_c \end{pmatrix} \begin{pmatrix} C \\ S \end{pmatrix} = 2\eta\varepsilon\Omega \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \dot{C} \\ \dot{S} \end{pmatrix}.$$

- We proceed to eliminate the frequency split.

Capacitor array

- Observe an annulus of thickness h surrounded by an array of electronic plates each at a small distance d from the cylindrical surface of the annulus. These plates, together with the cylindrical surface of the annulus approximate a "parallel plate capacitor" array:





Small potential differences

- Assume that small potential differences $\sqrt{\varepsilon}V_1$, $\sqrt{\varepsilon}V_2$, $\sqrt{\varepsilon}V_3$ and $\sqrt{\varepsilon}V_4$ are maintained between the plate and the disc for capacitors numbered one to four respectively, where we use the small parameter ε again to emphasise smallness.
- Assume that the other potential difference around the disc are $\frac{\pi}{2}$ periodic in the sense that capacitor number five has potential difference $\sqrt{\varepsilon}V_1$, capacitor number six has potential difference $\sqrt{\varepsilon}V_2$, et cetera.
- Consider the square of the electrical potential $\varepsilon V^2(\varphi)$ between the capacitors and the outer wall around the disc Let dA be an infinitesimal part of the surface area on the outer cylindrical wall.
- Assume $\varepsilon V^2(\varphi) = 0$ if there is no part of a plate covering the area dA while $\varepsilon V^2(\varphi) = \varepsilon V_1^2$ if dA is covered by the 1^{st}, 5^{th} , 9^{th} or 13^{th} plate, et cetera. An example of the situation is depicted in the following figure:

Small potential differences

- Assume that small potential differences $\sqrt{\varepsilon}V_1$, $\sqrt{\varepsilon}V_2$, $\sqrt{\varepsilon}V_3$ and $\sqrt{\varepsilon}V_4$ are maintained between the plate and the disc for capacitors numbered one to four respectively, where we use the small parameter ε again to emphasise smallness.
- Assume that the other potential difference around the disc are $\frac{\pi}{2}$ periodic in the sense that capacitor number five has potential difference $\sqrt{\varepsilon}V_1$, capacitor number six has potential difference $\sqrt{\varepsilon}V_2$, et cetera.
- Consider the square of the electrical potential $\varepsilon V^2(\varphi)$ between the capacitors and the outer wall around the disc Let dA be an infinitesimal part of the surface area on the outer cylindrical wall.
- Assume $\varepsilon V^2(\varphi) = 0$ if there is no part of a plate covering the area dA while $\varepsilon V^2(\varphi) = \varepsilon V_1^2$ if dA is covered by the 1^{st}, 5^{th} , 9^{th} or 13^{th} plate, et cetera. An example of the situation is depicted in the following figure:

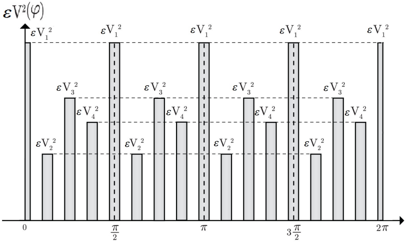
Small potential differences

- Assume that small potential differences $\sqrt{\varepsilon}V_1$, $\sqrt{\varepsilon}V_2$, $\sqrt{\varepsilon}V_3$ and $\sqrt{\varepsilon}V_4$ are maintained between the plate and the disc for capacitors numbered one to four respectively, where we use the small parameter ε again to emphasise smallness.
- Assume that the other potential difference around the disc are $\frac{\pi}{2}$ periodic in the sense that capacitor number five has potential difference $\sqrt{\varepsilon}V_1$, capacitor number six has potential difference $\sqrt{\varepsilon}V_2$, et cetera.
- Consider the square of the electrical potential $\varepsilon V^2(\varphi)$ between the capacitors and the outer wall around the disc Let dA be an infinitesimal part of the surface area on the outer cylindrical wall.
- Assume $\varepsilon V^2(\varphi) = 0$ if there is no part of a plate covering the area dA while $\varepsilon V^2(\varphi) = \varepsilon V_1^2$ if dA is covered by the 1^{st}, 5^{th} , 9^{th} or 13^{th} plate, et cetera. An example of the situation is depicted in the following figure:

Small potential differences

- Assume that small potential differences $\sqrt{\varepsilon}V_1$, $\sqrt{\varepsilon}V_2$, $\sqrt{\varepsilon}V_3$ and $\sqrt{\varepsilon}V_4$ are maintained between the plate and the disc for capacitors numbered one to four respectively, where we use the small parameter ε again to emphasise smallness.
- Assume that the other potential difference around the disc are $\frac{\pi}{2}$ periodic in the sense that capacitor number five has potential difference $\sqrt{\varepsilon}V_1$, capacitor number six has potential difference $\sqrt{\varepsilon}V_2$, et cetera.
- Consider the square of the electrical potential $\varepsilon V^2(\varphi)$ between the capacitors and the outer wall around the disc Let dA be an infinitesimal part of the surface area on the outer cylindrical wall.
- Assume $\varepsilon V^2(\varphi) = 0$ if there is no part of a plate covering the area dA while $\varepsilon V^2(\varphi) = \varepsilon V_1^2$ if dA is covered by the 1^{st}, 5^{th} , 9^{th} or 13^{th} plate, et cetera. An example of the situation is depicted in the following figure:

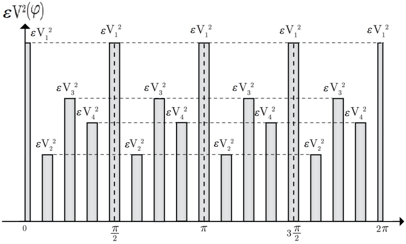
Fourier series



- Because of the periodicity involved with the potentials, we may determine a Fourier series for the function $V^2(\varphi)$ depicted in the figure as follows

$$V^2(\varphi) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos n\varphi + b_n \sin n\varphi \right).$$

Fourier series



- Because of the periodicity involved with the potentials, we may determine a Fourier series for the function $V^2(\varphi)$ depicted in the figure as follows

$$V^2(\varphi) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos n\varphi + b_n \sin n\varphi\right).$$

We use MATHEMATICA® to determine the a_n and b_n:

In[6]:= $a_{n\_} :=$

$$\frac{1}{\pi}\,\text{FullSimplify}\Big[\int_0^{\Delta\varphi} \varepsilon V_1^2 \text{Cos}[n\varphi]\, d\varphi + \int_{\frac{\pi}{8}-\Delta\varphi}^{\frac{\pi}{8}+\Delta\varphi} \varepsilon V_2^2 \text{Cos}[n\varphi]\, d\varphi + \int_{\frac{\pi}{4}-\Delta\varphi}^{\frac{\pi}{4}+\Delta\varphi} \varepsilon V_3^2 \text{Cos}[n\varphi]\, d\varphi +$$

$$\int_{\frac{3\pi}{8}-\Delta\varphi}^{\frac{3\pi}{8}+\Delta\varphi} \varepsilon V_4^2 \text{Cos}[n\varphi]\, d\varphi + \int_{\frac{\pi}{2}-\Delta\varphi}^{\frac{\pi}{2}+\Delta\varphi} \varepsilon V_1^2 \text{Cos}[n\varphi]\, d\varphi + \int_{\frac{5\pi}{8}-\Delta\varphi}^{\frac{5\pi}{8}+\Delta\varphi} \varepsilon V_2^2 \text{Cos}[n\varphi]\, d\varphi + \int_{\frac{3\pi}{4}-\Delta\varphi}^{\frac{3\pi}{4}+\Delta\varphi} \varepsilon V_3^2 \text{Cos}[n\varphi]\, d\varphi +$$

$$\int_{\frac{7\pi}{8}-\Delta\varphi}^{\frac{7\pi}{8}+\Delta\varphi} \varepsilon V_4^2 \text{Cos}[n\varphi]\, d\varphi + \int_{\pi-\Delta\varphi}^{\pi+\Delta\varphi} \varepsilon V_1^2 \text{Cos}[n\varphi]\, d\varphi + \int_{\frac{9\pi}{8}-\Delta\varphi}^{\frac{9\pi}{8}+\Delta\varphi} \varepsilon V_2^2 \text{Cos}[n\varphi]\, d\varphi + \int_{\frac{5\pi}{4}-\Delta\varphi}^{\frac{5\pi}{4}+\Delta\varphi} \varepsilon V_3^2 \text{Cos}[n\varphi]\, d\varphi +$$

$$\int_{\frac{11\pi}{8}-\Delta\varphi}^{\frac{11\pi}{8}+\Delta\varphi} \varepsilon V_4^2 \text{Cos}[n\varphi]\, d\varphi + \int_{\frac{3\pi}{2}-\Delta\varphi}^{\frac{3\pi}{2}+\Delta\varphi} \varepsilon V_1^2 \text{Cos}[n\varphi]\, d\varphi + \int_{\frac{13\pi}{8}-\Delta\varphi}^{\frac{13\pi}{8}+\Delta\varphi} \varepsilon V_2^2 \text{Cos}[n\varphi]\, d\varphi +$$

$$\int_{\frac{7\pi}{4}-\Delta\varphi}^{\frac{7\pi}{4}+\Delta\varphi} \varepsilon V_3^2 \text{Cos}[n\varphi]\, d\varphi + \int_{\frac{15\pi}{8}-\Delta\varphi}^{\frac{15\pi}{8}+\Delta\varphi} \varepsilon V_4^2 \text{Cos}[n\varphi]\, d\varphi + \int_{2\pi-\Delta\varphi}^{2\pi} \varepsilon V_1^2 \text{Cos}[n\varphi]\, d\varphi\Big]$$

Table[a_n, {n, 0, 14}]

Out[7]= $$\left\{\frac{8\,\Delta\varphi\,\varepsilon\,(V_1^2+V_2^2+V_3^2+V_4^2)}{\pi}, 0, 0, 0, \frac{2\,\varepsilon\,\text{Sin}[4\,\Delta\varphi]\,(V_1^2-V_3^2)}{\pi}, 0, 0, 0, \frac{\varepsilon\,\text{Sin}[8\,\Delta\varphi]\,(V_1^2-V_2^2+V_3^2-V_4^2)}{\pi}, 0, 0, 0, \frac{2\,\varepsilon\,\text{Sin}[12\,\Delta\varphi]\,(V_1^2-V_3^2)}{3\pi}, 0, 0\right\}$$

In[8]:= Table[b_n, {n, 1, 14}]

Out[8]= $$\left\{0, 0, 0, \frac{2\,\varepsilon\,\text{Sin}[4\,\Delta\varphi]\,(V_2^2-V_4^2)}{\pi}, 0, 0, 0, 0, 0, 0, 0, \frac{2\,\varepsilon\,\text{Sin}[12\,\Delta\varphi]\,(-V_2^2+V_4^2)}{3\pi}, 0, 0\right\}$$

Equations of motion

- Recall that $\omega^2 = \frac{I_2}{I_0} = \frac{2\int_p^q UVrdr}{\int_p^q (U^2+V^2)rdr}$ and examine the equations of motion that include mass imperfections and electrical potentials:

-
$$\begin{pmatrix}\ddot{C}\\ \ddot{S}\end{pmatrix} + \frac{1}{I_0}\begin{pmatrix} I_2 - \varepsilon k_1 - \varepsilon\left[k_2 + \rho_c I_2\right] & -\varepsilon\left[k_3 + \rho_s I_2\right] \\ -\varepsilon\left[k_3 + \rho_s I_2\right] & I_2 - \varepsilon k_1 + \varepsilon\left[k_2 + \rho_c I_2\right]\end{pmatrix}\begin{pmatrix}C\\ S\end{pmatrix}\begin{pmatrix}\dot{C}\\ \dot{S}\end{pmatrix}$$
$$= 2\eta\varepsilon\Omega\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$$

- where

$$k_1 = \frac{4\Delta\varphi hq\epsilon_0 U^2(q)}{\pi d^3}\left(V_1^2 + V_2^2 + V_3^2 + V_4^2\right)$$
$$k_2 = \frac{hq\epsilon_0\sin(4\Delta\varphi)U^2(q)}{\pi d^3}\left(V_1^2 - V_3^2\right)$$
$$k_3 = \frac{hq\epsilon_0\sin(4\Delta\varphi)U^2(q)}{}\left(V^2 - V^2\right)$$

Equations of motion

- Recall that $\omega^2 = \frac{I_2}{I_0} = \frac{2\int_p^q UVrdr}{\int_p^q (U^2+V^2)rdr}$ and examine the equations of motion that include mass imperfections and electrical potentials:

- $$\begin{pmatrix} \ddot{C} \\ \ddot{S} \end{pmatrix} + \frac{1}{I_0}\begin{pmatrix} I_2 - \varepsilon k_1 - \varepsilon\left[k_2 + \rho_c I_2\right] & -\varepsilon\left[k_3 + \rho_s I_2\right] \\ -\varepsilon\left[k_3 + \rho_s I_2\right] & I_2 - \varepsilon k_1 + \varepsilon\left[k_2 + \rho_c I_2\right] \end{pmatrix}\begin{pmatrix} C \\ S \end{pmatrix}\begin{pmatrix} \dot{C} \\ \dot{S} \end{pmatrix} = 2\eta\varepsilon\Omega\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

- where

$$k_1 = \frac{4\Delta\varphi hq\epsilon_0 U^2(q)}{\pi d^3}\left(V_1^2 + V_2^2 + V_3^2 + V_4^2\right)$$

$$k_2 = \frac{hq\epsilon_0 \sin(4\Delta\varphi) U^2(q)}{\pi d^3}\left(V_1^2 - V_3^2\right)$$

$$k_3 = \frac{hq\epsilon_0 \sin(4\Delta\varphi) U^2(q)}{}\left(V_2^2 - \right)$$

Equations of motion

- Recall that $\omega^2 = \frac{I_2}{I_0} = \frac{2\int_p^q UVrdr}{\int_p^q (U^2+V^2)rdr}$ and examine the equations of motion that include mass imperfections and electrical potentials:
- $$\begin{pmatrix}\ddot{C}\\ \ddot{S}\end{pmatrix} + \frac{1}{I_0}\begin{pmatrix} I_2 - \varepsilon k_1 - \varepsilon\left[k_2 + \rho_c I_2\right] & -\varepsilon\left[k_3 + \rho_s I_2\right] \\ -\varepsilon\left[k_3 + \rho_s I_2\right] & I_2 - \varepsilon k_1 + \varepsilon\left[k_2 + \rho_c I_2\right]\end{pmatrix}\begin{pmatrix}C\\ S\end{pmatrix}\begin{pmatrix}\dot{C}\\ \dot{S}\end{pmatrix} = 2\eta\varepsilon\Omega\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$$
- where

$$k_1 = \frac{4\Delta\varphi hq\epsilon_0 U^2(q)}{\pi d^3}\left(V_1^2 + V_2^2 + V_3^2 + V_4^2\right)$$
$$k_2 = \frac{hq\epsilon_0 \sin(4\Delta\varphi)U^2(q)}{\pi d^3}\left(V_1^2 - V_3^2\right)$$
$$k_3 = \frac{hq\epsilon_0 \sin(4\Delta\varphi)U^2(q)}{}\left(V_?^2 - V_?^2\right)$$

Negating mass imperfection

- Because $k_2 \propto \left(V_1^2 - V_3^2\right)$ and $k_3 \propto \left(V_2^2 - V_4^2\right)$, if we arrange capacitor voltage so that

$$\varepsilon\left[k_2 + \rho_c I_2\right] = 0 \quad \& \quad \varepsilon\left[k_2 + \rho_c I_2\right] = 0,$$

- then the equations of motion reduce to

$$\begin{pmatrix} \ddot{C} \\ \ddot{S} \end{pmatrix} + \frac{I_2 - \varepsilon k_1}{I_0} \begin{pmatrix} C \\ S \end{pmatrix} = 2\eta\varepsilon\Omega \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \dot{C} \\ \dot{S} \end{pmatrix}.$$

- Compare this with the equations of motion of an *ideal annular gyroscope:*

$$\begin{pmatrix} \ddot{C} \\ \ddot{S} \end{pmatrix} + \frac{I_2}{I_0} \begin{pmatrix} C \\ S \end{pmatrix} = 2\eta\varepsilon\Omega \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \dot{C} \\ \dot{S} \end{pmatrix}$$

where

$$\omega^2 = \frac{I_2}{I_0} = \frac{2\int_p^q UVrdr}{\int^q \left(U^2 + V^2\right) rdr}$$

Negating mass imperfection

- Because $k_2 \propto \left(V_1^2 - V_3^2\right)$ and $k_3 \propto \left(V_2^2 - V_4^2\right)$, if we arrange capacitor voltage so that

$$\varepsilon\left[k_2 + \rho_c I_2\right] = 0 \quad \& \quad \varepsilon\left[k_2 + \rho_c I_2\right] = 0,$$

- then the equations of motion reduce to

$$\begin{pmatrix}\ddot{C}\\ \ddot{S}\end{pmatrix} + \frac{I_2 - \varepsilon k_1}{I_0}\begin{pmatrix}C\\ S\end{pmatrix} = 2\eta\varepsilon\Omega\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}\begin{pmatrix}\dot{C}\\ \dot{S}\end{pmatrix}.$$

- Compare this with the equations of motion of an *ideal annular gyroscope:*

$$\begin{pmatrix}\ddot{C}\\ \ddot{S}\end{pmatrix} + \frac{I_2}{I_0}\begin{pmatrix}C\\ S\end{pmatrix} = 2\eta\varepsilon\Omega\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}\begin{pmatrix}\dot{C}\\ \dot{S}\end{pmatrix}$$

where

$$\omega^2 = \frac{I_2}{I_0} = \frac{2\int_p^q UVrdr}{\int_p^q \left(U^2 + V^2\right) rdr}$$

Negating mass imperfection

- Because $k_2 \propto \left(V_1^2 - V_3^2\right)$ and $k_3 \propto \left(V_2^2 - V_4^2\right)$, if we arrange capacitor voltage so that

$$\varepsilon\left[k_2 + \rho_c l_2\right] = 0 \quad \& \quad \varepsilon\left[k_2 + \rho_c l_2\right] = 0,$$

- then the equations of motion reduce to

$$\begin{pmatrix} \ddot{C} \\ \ddot{S} \end{pmatrix} + \frac{l_2 - \varepsilon k_1}{l_0} \begin{pmatrix} C \\ S \end{pmatrix} = 2\eta\varepsilon\Omega \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \dot{C} \\ \dot{S} \end{pmatrix}.$$

- Compare this with the equations of motion of an *ideal annular gyroscope:*

$$\begin{pmatrix} \ddot{C} \\ \ddot{S} \end{pmatrix} + \frac{l_2}{l_0} \begin{pmatrix} C \\ S \end{pmatrix} = 2\eta\varepsilon\Omega \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \dot{C} \\ \dot{S} \end{pmatrix}$$

where

$$\omega^2 = \frac{l_2}{l_0} = \frac{2\int_p^q UVrdr}{\int_p^q \left(U^2 + V^2\right) rdr}$$

Negative stiffness

- Hence, the capacitors have produced a gyroscope with mass imperfections that behaves "ideally" and is vibrating with a slightly reduced angular rate

$$\omega^* = \sqrt{\frac{I_2 - \varepsilon k_1}{I_0}}$$

where $-\varepsilon k_1$ reduces the stiffness integral I_2 and is called **negative stiffness**. The term $-\varepsilon k_1$ will never disappear because $k_1 \propto \left(V_1^2 + V_2^2 + V_3^2 + V_4^2\right)$.

- An annular vibratory gyroscope manufactured by including this array of capacitors and manipulating them appropriately will be able to utilise Bryan's factor η to determine the rotation rate $\varepsilon\Omega$ of the vehicle in which it is mounted using the formula

$$\varepsilon\Omega = \frac{\text{Rate of rotation of the vibrating pattern of the gyroscope}}{\eta}.$$

Negative stiffness

- Hence, the capacitors have produced a gyroscope with mass imperfections that behaves "ideally" and is vibrating with a slightly reduced angular rate
$$\omega^* = \sqrt{\frac{I_2 - \varepsilon k_1}{I_0}}$$
where $-\varepsilon k_1$ reduces the stiffness integral I_2 and is called **negative stiffness**. The term $-\varepsilon k_1$ will never disappear because $k_1 \propto \left(V_1^2 + V_2^2 + V_3^2 + V_4^2\right)$.

- An annular vibratory gyroscope manufactured by including this array of capacitors and manipulating them appropriately will be able to utilise Bryan's factor η to determine the rotation rate $\varepsilon\Omega$ of the vehicle in which it is mounted using the formula
$$\varepsilon\Omega = \frac{\text{Rate of rotation of the vibrating pattern of the gyroscope}}{\eta}.$$