Otto Wurnig, University of Graz (Austria)

Using TI-92 in the 9th-Grade of Austrian Grammar Schools - Hypotheses, Experiences, Results, Problems.

In **1997/98** thirty classes of the 9th-Grade in grammar schools participated in the Austrian TI-92 project. (WURNIG, 1998). After a first phase of learning how to use the TI-92 in a successful way, the teachers of the project had to carry out a sequence of lessons in the same way in every semester. Both sequences began with a test to control the knowledge of the use of the TI-92. The goal was to find out with which exercises it would be possible to get comparable conditions in all classes.

Test I

Compute the following examples on your TI-92 and enter your minutes on the lines indicated. (IL is the TI-92 input line, OL the TI-92 output line):

(1) *Substitute the number 7 for x in the term 12x-20*:

IL: *12x-20 | x=7* OL: *64*

(2) *Multiply the following terms:* (3x²+7) (2x-3)

IL: *EXPAND((3x²+7).(2x-3))* OL: *6x³-9x²+14x-21*

(3) *Convert the result of (2)into the input of (2):*

IL: *FACTOR(6x³-9x²+14x-21)* OL: *(3x²+7).(2x-3)*

(4) *Solve the equation* 3x-7 = 4(4x+2) *using the TI-command SOLVE:*

IL: *SOLVE(3x-7=4.(x+2),x)* OL: *x=-15/13*

(5) Tabulate the linear function y = 2.43∗x - 5.36 *in the interval [3,5] with the stepsize 1*:

IL: *2.43x - 5.36 → y1(x)* Table:

x	y1(x)
3	*1.93*
4	*4.36*
5	*6.79*

Calculate the longest side in the rectangular triangle (a=12.3 and b=16.7):

IL: _$c^2=12.3^2+16.7^2$_ OL: _$c^2=430.18$_
IL: _$\sqrt{(c^2=430.189)}$_ OL: _$|c|=20.7408$_

(7) *Solve the following system of linear equations in the window GRAPH using the TI-command INTERSECTION.*

1. equation: 3x + 5y = -5 2. equation: 2x - 3y = 22

IL: y1(x) = _-(3x+5)/5_ window GRAPH:
y2(x) = _2.(x-11)/3_ xc: _5_ yc: _-4_

(8) *Solve the linear equation 8x - 1 -2(5x - 4) =0 in the window GRAPH using the TI-command INTERSECTION (Intersect the graph of the linear function f(x) with the x-axis):*

IL: y1(x) = _-2x+7_ window GRAPH:
y2(x) = _0_ xc: _3.5_ yc: _0_

Statistics of the student achievements				
19 classes	**right ≥ 80%**	**right > 50%**	**right ≤ 50%**	**sum**
Pretest I	**61%**	**27%**	**12%**	**100%**

In the following lessons the teachers had to observe, which ways of solving a new problem students were taking when using a computer algebra system. The sequence ended with a final test.

In the **first lesson of the sequence** the thirty teachers of the 9th form had to find out the various methods the students were taking with the TI-92, when they had **to solve a quadratic equation for the first time**. As the co-ordinator I sent the following example to all research teachers:

A rectangular triangle has the sides x, x+3, x+6. Compute x!
Solve the equation you get in three ways at least. Document your ways of solution in such a way, that anybody can comprehend the most important steps, you did.

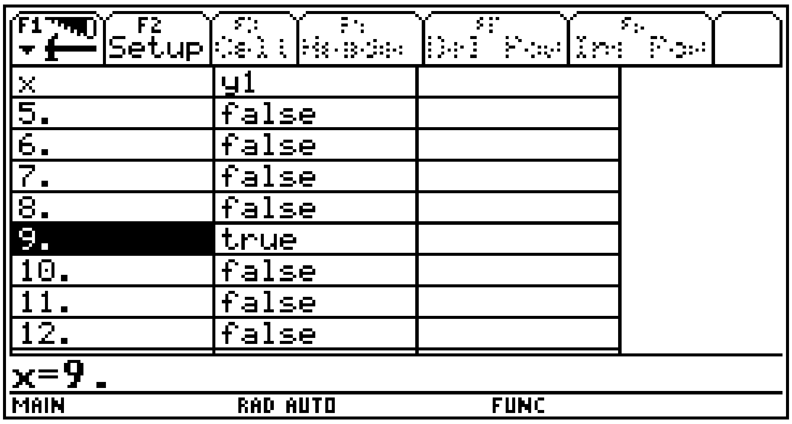
Different ways	**SOLVE**	**Graph**	**Table**	**per hand**	**FACTOR**
percent	93%	50%	39%	4%	3%
number of ways	**0 way**	**1 way**	**2 ways**	**3 ways**	**more than 3 ways**
percent	7%	35%	18%	30%	10%

Studying the table you can see that nearly all students (93%) used the command **SOLVE**. The students first entered the equation **$(x+6)^2 = (x+3)^2+x^2$** and then most of them took the TI-command SOLVE to get the result.

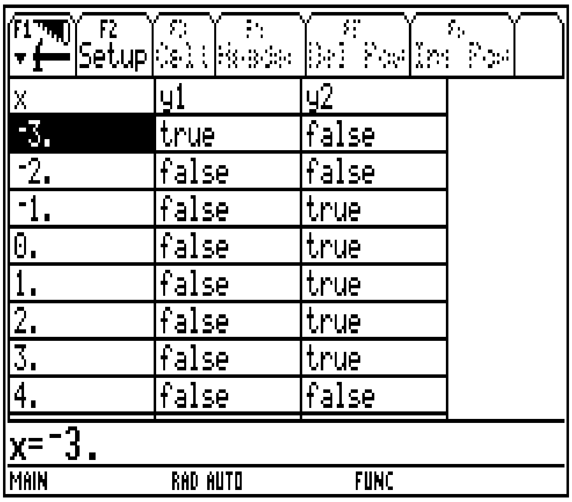
As hoped for, 58% of the students found at least a second way (table or graph).

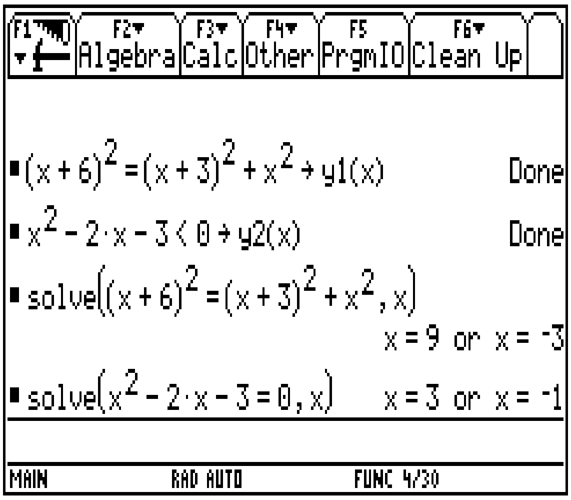
It was surprising though, which variants of right ways were found by the students. The **following two variants of solution** strongly influenced the further mathematical education.

Some students saved the equation $(x+6)^2=(x+3)^2+x^2$ without changing it as function y1(x). In this case the table showed *true* for *x=9*.

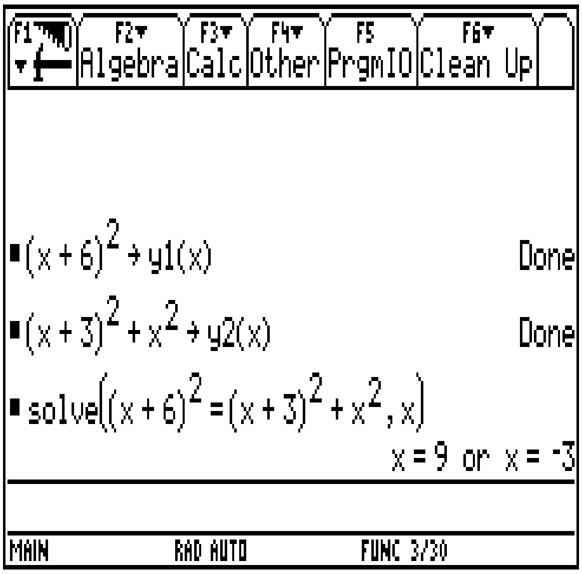


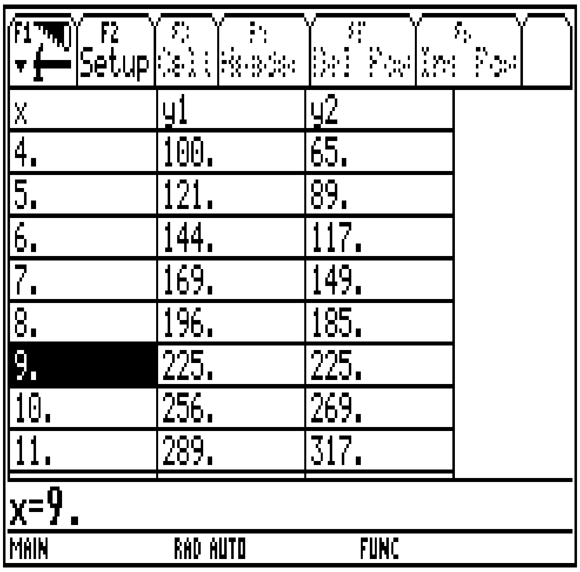
This way infrequently leads to the right solution with equations, but it is a very good way for inequalities!



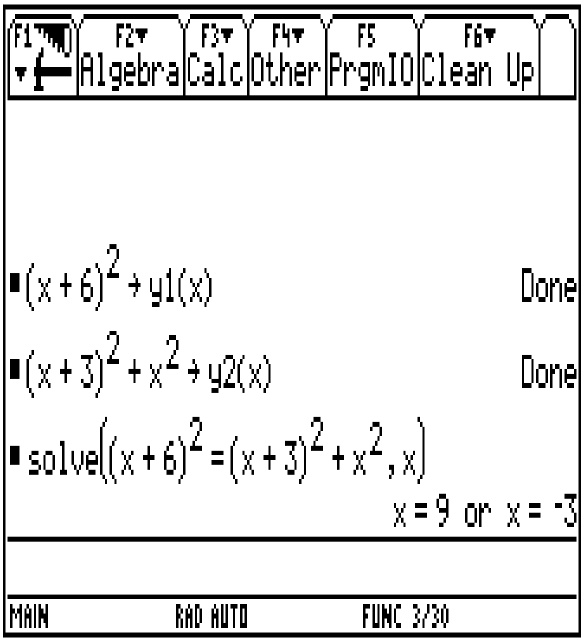


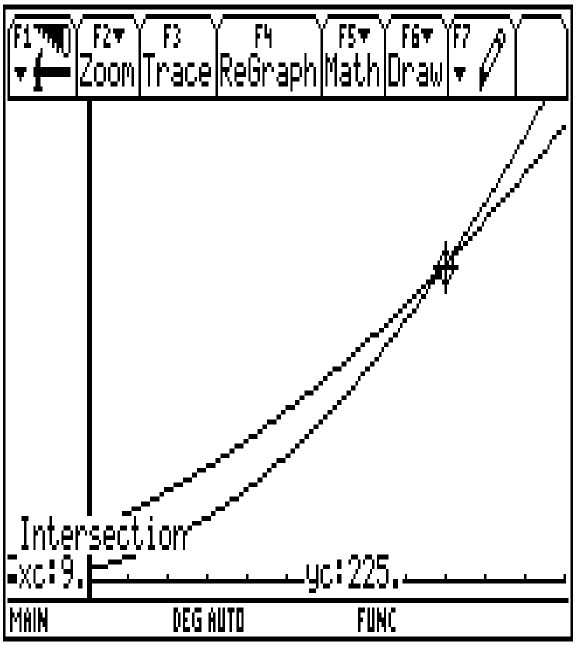
Some students saved the expression **(x+6)²** under **y1(x)** and the expression **(x+3)²+x²** under **y2(x)** and they found that the ***value of y*** *is the same* (=225) in both tables when ***x=9***. The students found this way in analogy to the way of solution for a system of linear equations with two variables!



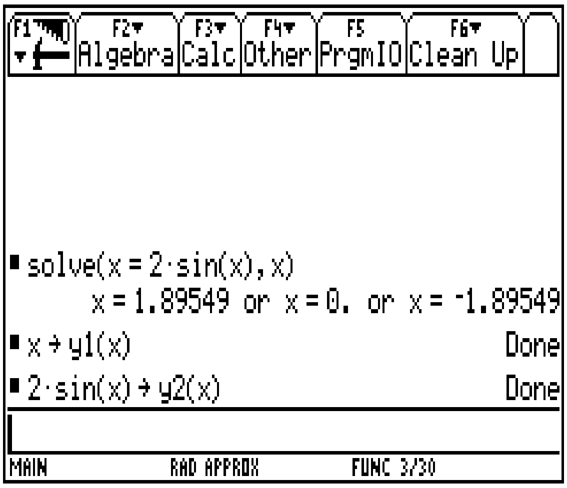


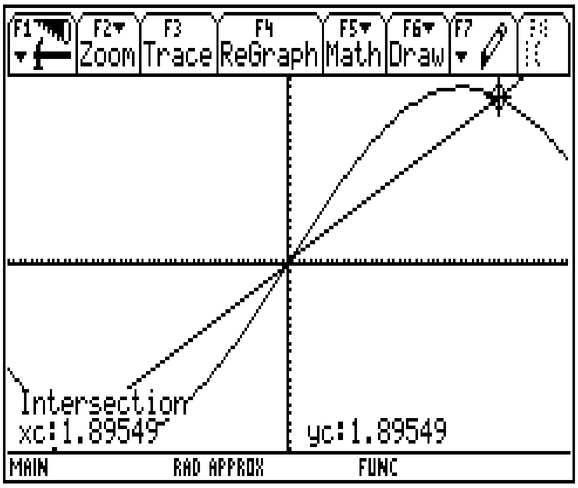
The way with two tables produces the basis for the right understanding of the graphic way for the intersection of two curves (TI-command INTERSECTION).





This graphic way for the intersection of two curves (TI-command INTERSECTION) is very important for the solution of algebraic equation such as **x = 2 . sin(x)** in the 10^{th} form!





After having done a sequence of lessons of **quadratic equations and quadratic functions,** the teachers had to give the following example as final test:

Solve the quadratic equation $x^2 + 3x - 15 = 0$ **in four different methods (one without the TI-92 and three with the TI-92)!**

Every teacher had to state with which different methods the students had solved the problem given and then had to write a short report. In my class I did not only want to find out **which ways of solution the students would choose**, I also wanted to know which of the ways they chose **in first** and which **in second position**.

<u>Distribution of the methods of my 25 students</u>:

22x **SOLVE** (18x in 1st position),

21x **the formula for solution** (5x in 1st , 14x in 2nd position, 4x with mistakes in calculation),

*9x **graphical method,** 8x **INTERSECTION**, 3x **Table** (counted only with stepwise 0.1),*

*6x **Vieta** (can be regarded only as a check over in this case)*

*2x **quadratic complement,** 2x **FACTOR***

<u>Distribution according to the number of ways of solution</u>:

number of ways	**0 way**	**1 way**	**2 ways**	**3 ways**	**≥ 4 ways**
number of students	**0**	**5**	**5**	**3**	**12**

<u>Distribution according to the ways carried out successfully</u>:

number of ways	**0 way**	**1 way**	**2 ways**	**3 ways**	**≥ 4 ways**
number of students	**1**	**5**	**6**	**2**	**11**

Considering the instruction the students had been given I was surprised to see that the ***method table*** *(stepwise 1 was not counted) and the method using the command* ***FACTOR*** *were so little used.*

After all the reports had been collected and studied, it could be proved that ***the use of the TI-92 really led to a greater variation of ways of solution*** *when approaching a problem for in some classes students had found* ***up to nine different ways of solution****.*

The second sequence concerning **analytic geometry** was taught **at the end of the year**.

The teachers had to observe how the students liked to solve **analytic problems with modules offered by the TI-92** such as norm, unitv, dotp and crossp or if they also tried to find functions of their own. The results of the observations were very different and presumably depended on whether teachers were ready to apply the modular method in a useful way in their lessons.

The ***first pretest*** *was carried out in* ***21 classes*** *whereas the* ***second pretest*** *was carried out in* ***14 classes*** *only.* ***Two reasons*** *probably account for that:*

- *The* ***second pretest*** *was carried out* ***after having finished teaching analytic geometry.*** *That means it was carried out at the end of term and so many teachers were too busy with assessing the work of their students.*
- ***Solving problems with modular methods*** *was new for those research teachers not engaged in the DERIVE project in 1993/94 and for those who had never used little computer programs in mathematical education so far.*

Test II

Compute the following examples on your TI-92 and enter your minutes on the lines indicated (IL is the TI-92 input line, OL the TI-92 output line:

Save the points A, B, P as follows: [0;-1] → a : [4;2] → b : [-5;4] → p

(1) *Compute the vector AB*:

IL: *b - a* OL: *[4;3]*

(2) *Calculate the length of the vector AB using the TI-function NORM*:

IL: norm(b-a) OL: 5

(3) *Set up the parametric equation of the line AB and save it under g:*

IL: *[x;y]=a+t.(b-a) → g* OL: *[x=4t; y=3t-1]*

(4) *Save a vector which is orthogonal to the vector AB under n:*

IL: *[-3;4] → n* OL: *[-3;4]*

(5) *Eliminate the parameter in equation (3) using the inner product of g and n*:

IL: *dotp(g,n)* OL: *-3x+4y=-4*

(6) *Compute the midpoint of the line segment AB and save it under m:*

IL: *(a+b)/2 → m* OL: *[2;1/2]*

(7) *Set up the parametric equation of the perpendicular bisector of AB*:

IL: *[x;y]=m+t.n* OL: *[x=-3t+2;y=4t+1/2]*

(8) *Compute the unit vector of AB with the formula taught and then compare it with the result of the TI-function UNITV*:

IL: *(b-a)/norm(b-a)* OL: *[4/5;3/5]*

IL: *unitv(b-a)* OL: *[4/5;3/5]*

(9) *Calculate the perpendicular distance of the point P from the line AB with the formula taught:*

IL: *dotp(p-a,unitv(n))* OL: *7*

(10) *Compute the direction vector of the angle bisector of (AB,AP) to 3 digits:*

IL: *unitv(b-a)+unitv(p-a)* OL: *[0,093;-1,307]*

The following table shows the achievement of the 14 classes having participated in pretest I and II:

Statistics of the student achievements

14 classes	right ≥ 80%	right > 50%	right ≤ 50%	sum
Pretest I	61%	27%	12%	100%
Pretest II	53%	36%	11%	100%

The comparison of the student achievements shows clearly that the **number of the weak students was constant**, whilst the number of the better students slightly sank with the increasing difficulty of the problems.

After the correction of pretest II the 14 research teachers asked their students to solve the following example. The set up of the working sheet was **to motivate the teachers and students to work with modules**.

Use the TI-92 when solving the following example in a skilful way and document the most important steps of yout TI-92 input and output.

Triangle ABC: A=[-3;-5] B=[9;4] C=[-3;9]

a) Compute the intersection point of two perpendicular bisectors.

b) Compute the intersection point of two angle bisectors.

c) Test if the point P=[3;-13] is lying on the line IU.

(I=[1;3] and U=[9/8;2])

First sketch a diagram and then carry out the example with the help of the formulas you have been taught.

At the beginning of part a) and c) on the working sheet the following suggestion as to how the input could be stored was given:

a) Input:	*[-3;-5]* $\rightarrow$ *a* :	*[9;4]* $\rightarrow$ *b* :	*[-3;9]* $\rightarrow$ *c*
b) Input:	*[3;-13]* $\rightarrow$ *p* :	*[1;3]* $\rightarrow$ *i* :	*[9/8;2]* $\rightarrow$ *u*

Statistics				
14 classes	**(nearly) right**	**≥ ½ right**	**few/wrong**	**sum**
part a	**59%**	**21%**	**20%**	**100%**
part b	**53%**	**15%**	**32%**	**100%**
part c	**59%**	**7%**	**34%**	**100%**
a) b) and c)	**43%**	**32%**	**26%**	**100%**

All 14 research teachers, who had assessed the pretest II, sent their results. Many students at least tried to sketch out the beginning of a diagram but only few students per class tried to make a plan with formulas before beginning to compute as suggested.

The following reasons explain that the results in b) were not quite so good as in a) and c):

The computing of the intersection point of two angle bisectors is a difficult task and takes much time. Therefore some students left it out at first and did not finish it later on or when taken as second problem it took the students so much time that there was no time left to finish part c).

In the report of the **DERIVE-Project** H. HEUGL writes in **1996:**

> „It would be i*deal if every student had a portable CAS-calculator, which could be linked up with the CAS in a computer lab, in his school bag.“*

When the TI-92 was presented in Austria many research teachers of the Austrian CAS I project thought that the TI-92 was such a portable CAS-calculator. In the **Austrian CAS II Project (TI92-Project) 1997/98** the students of all the seventy research teachers wrote their tests in mathematics with the TI-92.

At their final meeting in August 1998 they collected their most important and sometimes unexpected results: (LECHNER/WURNIG, 1999)

- for solving problems it is very important **not always to insist on the use of the TI-92**.
- **difficult decision**: What **minimum knowledge of TI-commands** is an absolute must?
- **difficult decision**: What is to be the **minimum knowledge** in mathematics **without** TI-92?

Solve 2 formulas per hand according to the indicated variables!
a) v = ? (no double fraction in the result!)
$y = v\cdot t - \frac{g\cdot t^2}{2}$
b) h = ? (no double fraction in the result!)
$e = m\cdot g\cdot h + \frac{m\cdot v^2}{2}$

- students find **new ways with the TI-92** → **more work for the teacher**

- **modules** and **programs** are a good chance **for good students**

  → **a new problem for bad students**.

Calculate the perpendicular distance of the point P from the line AB with the formula taught:
[0;-1] → a : [-3;4] → n : [-5;4] → p
IL:___*dotp(p-a,unitv(n))*_______ OL:__________*7*_______________

- the problems have to be **more goal oriented** → **text longer** instead of shorter.

1) The curve, which a flying object makes in the air is defined through the graph of the following function: **$f(t) = 45 + 20t - 5t^2$.** h means the height and t the time. **Draw the curve** of the flying object **in the (t, h) - co-ordinates system** by using of the TI-92 in your test book!
a) **At what time** does the object reach the highest point of the trajectory and when does it hit the ground (h=0)?
b) **In which height** does the trajectory start and when does the object reach the same height again? **Explain your answers!**

- TI-92 has no floppy → **much documentation** in test book, therefore **fewer examples**.

The point* P = [-8;-3] *is to be reflected on the line g: 3x+2y+4=0 *(<u>Sketch a diagram</u> on the working sheet!)* ***What are the co-ordinates of the reflected point?***
***(a) F**irst **s**olve the problem **geometrically** in your test book!*
***(b)** Solve the problem **in an algebraic way with the methods of the analytic geometry** on the working sheet!*
<u>Write down your plan in the left column and document the important interim results in the right column</u>! Do the calculations without the TI-92 on the flip side!

Left: steps of the plan!	**right: TI-92 documentation!**
P	[-8;-3] → p
g1: 3x + 2y + 4 = 0	[3;2] → n
g2 = g(∍ P, ⊥ g1)	X = P + t . n
	[x = 3t - 8, y = 2t - 3]
g1 ∩ g2 = {S}	3.(3t - 8) + 2.(2t - 3)+4=0
	SOLVE(ans(1), t) t = 2
	S = [-2;1]
S=(P + P*)/2 → P* = 2S - P	P* = [4; 5]

Literature:

HEUGL, H./KLINGER, W./LECHNER, J.: Mathematikunterricht mit Computeralgebra-Systemen. Addison-Wesley, Bonn 1996.

LECHNER, J./WURNIG, O.: Schularbeiten, 5. Klasse. In: KLINGER, W.(Hg.): Der Mathematikunterricht im Zeitalter der Informationstechnologie, ACDCA, 1999.

WURNIG, O.: 5. Klasse, Beobachtungsfenster 1 und 2; In: KLINGER, W.(Hg.): Der Mathematikunterricht im Zeitalter der Informationstechnologie, ACDCA, 1998.