



**Vienna International Symposium  
on Integrating Technology  
into Mathematics Education**

**July 10-13, 2002**

**Vienna, Austria**

# **Conference Program**

# VISIT-ME-2002 / Conference Program

..... Strudlhofgasse .....			..... Boltzmanngasse .....			..... Währingerstraße .....			..... other .....
HS Ph	HS 1		Sem Ph	HS 3	HS 4	PC-Lab A	Praktikum A	HS 6	

## Wed Jul 10

9:30-9:45		<b>Conf. Opening</b>
10:00-10:25	Beaudin (d013)	Torres-Skoumal (a072)
10:30-10:55		
11:00-11:25	Pence (d062)	Engelbrecht/Harding (a074)
11:30-11:55		
12:00-12:25	Urrego (d005)	Flynn (a032)
12:30-12:55	Hector (d046)	
13:00-13:25	McRae (a023)	Leinbach (d050)
13:30-13:55		
14:00-14:25	Jones (a068)	Quesada (d028)
14:30-14:55		
15:00-15:25	Abramson (a001)	<b>Baumann (d067)</b>
15:30-15:55		

<b>Schlöglhofer (d056)</b>	Biryukov (d026)		Oostenbroe (a064)	Stolyarevsk (a044)
	Townsley (d059)	Suwannaprasert (a005)	Workshop	Workshop
<b>Lüke-Rosendahl (d000)</b>	Schonefeld (d065)	Williams (a060)	MS Office	Prolog
	Pagon (d033)	Harris (a021)		
<b>Schmidt/Köhler (d036)</b>		Graubner (a048)	Pemberton (a066)	Mariz (a059)
<b>Schlichthorn (d058)</b>	Maturo (a081)	Dimitrova/Georg (d006)	Workshop	Workshop
<b>Neuper (a050)</b>	Workshop	Galan Garcia (d011)	Maple	GeoSketchpad
<b>Griesmayer (a027)</b>	TI-92+	Mann (d047)		
<b>Krempler (a052)</b>	Bowers (d043)	Justan (d023)	Wiesenbau (d053)	Goroneskul (a054)
<b>Klemenz (a053)</b>	Workshop	Joubert/Fay (d009)	Workshop	Workshop
<b>Heinrich (a080)</b>	TI-92+	Balderas (d042)	Derive	Maple
	Watanabe (d012)	Galan Garcia (d010)		

16:30-17:30

17:30-18:00

18:00-19:00

..... Vienna Univ .....

<b>Welcome Reception</b>
<b>Conference Welcome</b>
<b>Keynote Buchberger</b>

## Thu Jul 11

9:00-10:00	<b>Keynote Rich</b>	
10:00-10:25		Lechner (d037)
10:30-10:55	Kokol-Voljc (a091)	
11:00-11:25	Peschek (a086)	Ethells (d001)
11:30-11:55		
12:00-12:30	Schneider (a085)	Cretchley (d031)

<b>Aspetsberger/Aspetsber (d054)</b>	Maturo (a088)	Galan Garcia (d019)	<b>Baumann (d064)</b>
	Workshop		Workshop
<b>Urban-Woldron (d061)</b>	TI-89/92+	Albano (d048)	Derive
	Klinger (a010)		
<b>Führer-Nagy (d066)</b>	Cousquer (a077)	Cunningham (d045)	

14:00-\*\*\*

**Excursion  
(Vienna, Wachau,  
Burgenland)**

**Fri Jul 12**

9:00-10:00	<b>Keynote Weigand</b>						
10:00-10:25		Zarzycki (d032)		Malitte (a035)	Herget (a030)	Connors/Connors (d002)	Schofield (d017)
10:30-10:55	Tonisson (a058)				Workshop TI-92+	Hitt (d008)	Workshop Derive
11:00-11:25	Baldin (a034)	Roanes-Lozano (d015)		Kadunz (a084)	Pikalova (a069)		
11:30-11:55							
12:00-12:25	Hockman (a029)	Biryukov (d027)		Elschenbroich (a004)	Mráz (a039)	Ahlander (d007)	Leinbach (d049)
12:30-12:55					Flores (d004)	Gachkov (d020)	Workshop
13:00-13:25	Peck (a067)	Lagrange (a061)		Wurnig (d022)	Workshop TI-92+	Yaacob (a046)	TI Interactive!
13:30-13:55	Schultz (a043)					Johnson (a028)	
14:00-14:25	Schofield (d018)	Bouhineau (a042)	Poster session	Schröfel (d040)	Cunningham (d044)	Fu (a017)	Böhm (d038)
14:30-14:55	Biryukov (d025)				Workshop TI-89/92+	Watanabe (a016)	Workshop Derive
15:00-15:25	Sjöstrand (d041)	Klincsik (a020)		Brenner (a024)		Petraskova/Klufa (a041)	
15:30-15:55					Keunecke (d068)	Elschenbroich (a003)	
16:00-16:25	Jeffrey/Beaud (d057)	Ball (a089)		Karnel (a031)	Peterburgsky (a092)	Schmidt (a087)	Olive (a045) WS
16:30-16:55	Anisiu (d024)	Jindal (a002)		Goldgruber (a051)	Oleinik (a076)	Kadijevich (a049)	GeoSketchpd

Maturo (a019)
Workshop TI-83+
Nixon (a014)
Workshop TI-83+

19:00-\*\*\*

*.. Vienna City Hall ...*

Banquet

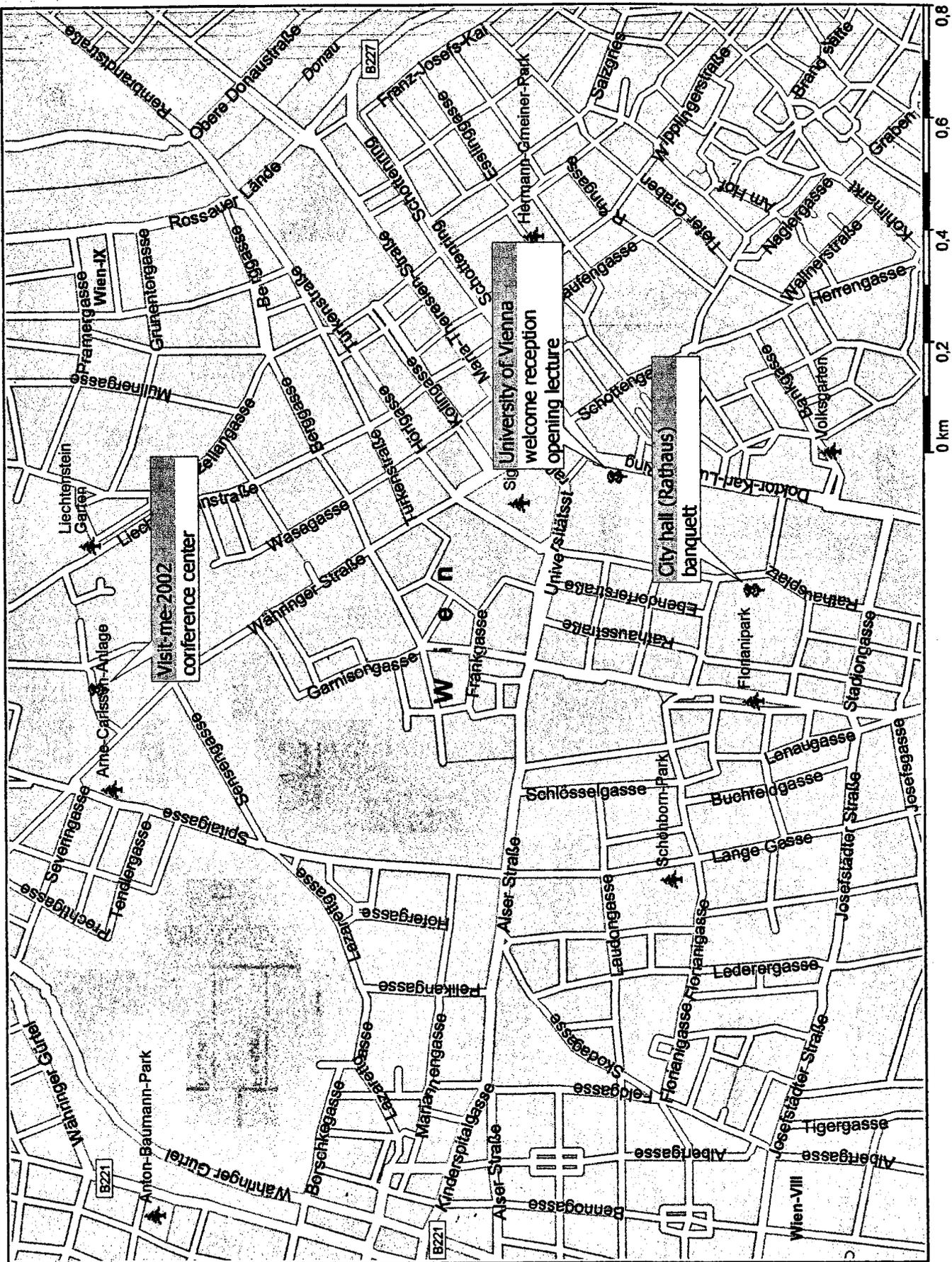
**Sat Jul 13**

9:00-10:00	<b>Keynote De Guzman</b>					
10:00-10:25		Pountney (a015)		Lehmann (a006)	Ockermann (d030)	Decker (a056)
10:30-10:55					Workshop TI-92+	Saunders (a070)
11:00-11:25		Pemberton (a065)		Himmelbauer (a007)		Golding (a036)
11:30-11:55	Noguera/Ndun (d029)				Rosing (d063)	Blyth (a057)
12:00-12:25	Openhaim (d051)	Neuper/Griesmayer/ Krem/Kar (a008)		Sarvari (a018)	Workshop TI-92+	Byelyavtsev (a063)
12:30-12:55						Bilousova (a062)
13:00-13:50	<b>Conf. Closing Farewell Recept.</b>					

Legend:

English  
German





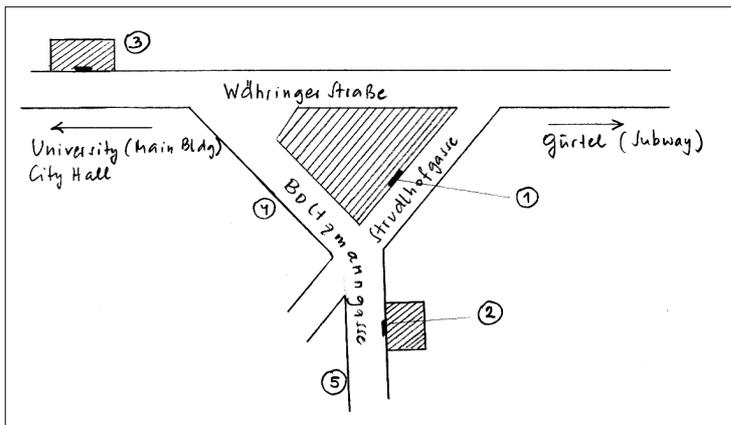
Visit-me-2002  
conference center

University of Vienna  
welcome reception  
opening lecture

City hall (Rathaus)  
banquet

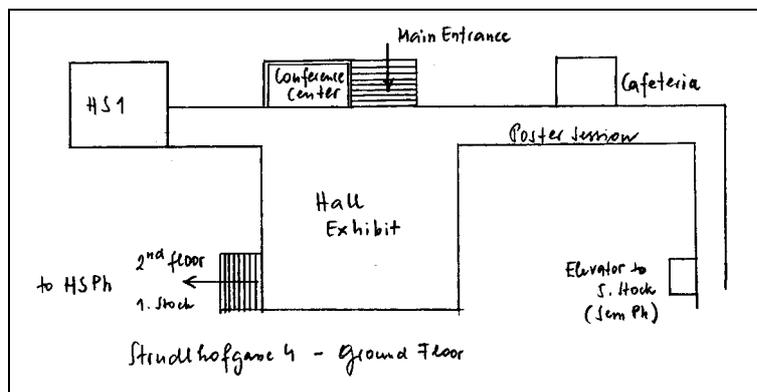
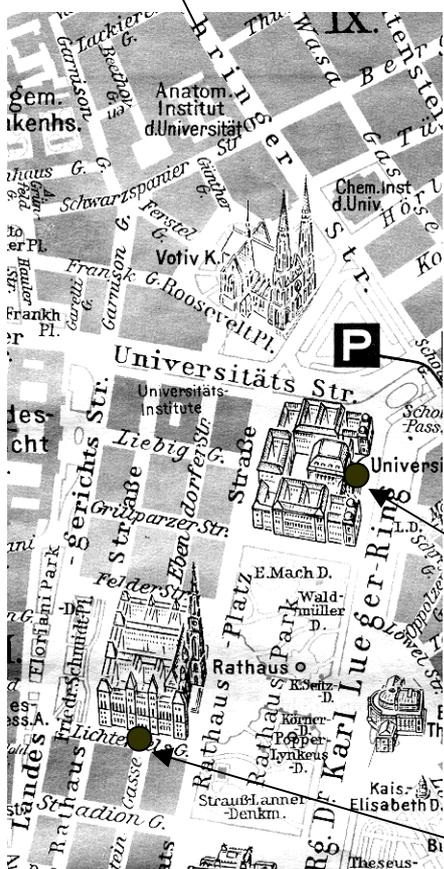
# Maps

- ① **Strudlhofgasse 4**  
Conference Center, Lecture Hall 1 (HS 1), Lecture Hall Physics (HS Ph), Seminar Room Physics (Sem Ph), Cafeteria, Poster Session, Exhibition
- ② **Boltzmannngasse 9**  
Lecture Halls 3 & 4 (HS 3, HS 4)
- ③ **Währinger Straße 17**  
PC-Lab A, Praktikum A, Lecture Hall 6 (HS 6)
- ④ **Hotel Arcotel Boltzmann**
- ⑤ **American Embassy**



Map of Mathematical Institute Area

Währinger Straße  
(for Mathematical Institute Area & VISIT-ME-2002)



Map of Mathematical Institute at „Strudlhofgasse 4“

The Mathematical Institute of the Vienna University is located in the 9<sup>th</sup> district "Alsergrund".

Entrance University  
(for Welcome Reception & Keynote Buchberger)

Entrance City Hall (Rathaus), Lichtenfelsgasse  
(for Banquet)

# VISIT-ME-2002

combines the

**7<sup>th</sup> ACDCA Summer Academy**

and the

**5<sup>th</sup> Int'l Derive & TI-89/92 Conference**

and is organized by the

**ACDCA (Austrian Center for Didactics of Computer Algebra)**

and the

**Institute of Mathematics at the University of Vienna.**

## Purpose of the Conference

In 1992 ACDCA (the Austrian Center for Didactics of Computer Algebra) started a conference series which has become a driving force in bringing technology, in particular computer algebra systems (CAS), into the classroom.

The conference series comprises two strands: the ACDCA Summer Academies, which are more oriented towards didactical questions connected with the use of technology for teaching and learning and the International Derive Conferences, which are more oriented towards concrete applications of Derive and similar tools for teaching in schools and at universities. In summer 2002, for the first time, the two events are held at the same time at the same place.

## History of the Conference

The past conferences were:

- 1992: 1<sup>st</sup> ACDCA Summer Academy, Krems, Austria
- 1993: 2<sup>nd</sup> ACDCA Summer Academy, Krems, Austria
- 1994: 1<sup>st</sup> Int'l Derive Conference, Plymouth, UK
- 1995: 3<sup>rd</sup> ACDCA Summer Academy, Honolulu, USA
- 1996: 2<sup>nd</sup> Int'l Derive Conference, Bonn, Germany
- 1997: 4<sup>th</sup> ACDCA Summer Academy, Sarö, Sweden
- 1998: 3<sup>rd</sup> Int'l Derive & TI-89/92 Conference, Gettysburg, USA
- 1999: 5<sup>th</sup> ACDCA Summer Academy, Gössing, Austria
- 2000: 6<sup>th</sup> ACDCA Summer Academy, Portoroz, Slovenia
- 2000: 4<sup>th</sup> Int'l Derive & TI-89/92 Conference, Liverpool, UK

## Conference Co-Chairs

Josef Böhm (nojo.boehm@pgv.at)  
Helmut Heugl (hheugl@netway.at)  
Bernhard Kutzler (b.kutzler@aon.at)  
Hans-Christian Reichel (reichel@radon.mat.univie.ac.at)

## Social Program Chair

Nancy Köstlbauer

## Technical Support Team

Helmut Achleitner  
Peter Nussbaumer

## Program Committee (ACDCA)

Vlasta Kokol-Voljc, Slovenia (Co-Chair)  
Bernhard Kutzler, Austria (Co-Chair)  
Brian Denton, UK  
Wilfried Herget, Germany  
Jean-Baptiste Lagrange, France  
Edith Schneider, Austria  
James Schultz, USA  
Marlene Torres-Skoumal, Austria  
Bernard Winkelmann, Germany

## Program Committee (Derive & TI-89/92)

Josef Böhm, Austria (Co-Chair)  
Carl Leinbach, USA (Co-Chair)  
Michel Beaudin, Canada  
Josef Lechner, Austria  
Francisco Puerta, Canary Islands, Spain  
Eugenio Roanes, Spain  
Theresa Shelby, USA  
David Sjöstrand, Sweden  
Hubert Weller, Germany  
Johann Wiesenbauer, Austria

## Conference Sponsors

Austrian Ministry of Education  
Texas Instruments  
Vienna Convention Bureau

## Keynote Presentations

Bruno [Buchberger](#) (University of Linz, Austria), [Bruno.Buchberger@risc.uni-linz.ac.at](mailto: Bruno.Buchberger@risc.uni-linz.ac.at)

### Teaching Without Teachers?

- By algorithms, problem solving in more and more areas of mathematics is automated.
- Even inventing and proving mathematical theorems can be automated more and more.
- The success of mathematical software systems and systems for supporting teaching suggests that also math teaching can be highly automated.
- Is it really possible and desirable to eliminate human math teachers?
- In the talk, we will analyze this question. In contrast to the expectations and fears raised by a superficial view on the natural flow of science towards automation, we will argue that the role of the human teacher in the (math) teaching process will become more important, more challenging, more interesting and more rewarding than ever. The argument is mainly based on the insight that mathematical evolution is an open process that proceeds in spirals of self-applications. What is creative in one loop through the spiral becomes trivial in the next loop leaving room for more creativity on a higher level.

We will discuss implications of this view on the future of math teaching technology and math didactics including the future education of math teachers. The talk will be accompanied by a few demos within the Theorema system

*(Wed July 10, 18:00, Festival Hall of University of Vienna)*

Miguel [de Guzman](#) (Universidad Complutense de Madrid, Spain), [mdeguzman@bitmailer.net](mailto: mdeguzman@bitmailer.net)

### Experiment, Conjecture, and Proof in Geometry with DERIVE

The core of the ordinary work of the research mathematician consists essentially in experimenting, conjecturing and proving. The main task of the one who tries to learn what mathematics is and how to become in any degree a mathematician should be to look at the mathematician's way to act when he or she is at work and to try to imitate and follow the same steps. Therefore teaching, and learning, mathematics should be essentially a continuously interactive occupation between the three main elements involved: teacher, learner, mathematics.

Experimentation in mathematics has become, through the presence of the recent technological instruments something very different, much easier, more interesting and more fruitful than it never before has been. One can experiment numerically, algebraically, visually,... with such an ease as it was never dreamed of a few years ago. And the possibilities of experimentation are improving very quickly and will continue doing so in the future. Our abilities to conjecture the presence of very deep and rich mathematical interrelations are in this way becoming greatly expanded through the simple exercise of improving and refining more and more our experiments until we can be practically sure that the theorems we guess through our repeated experiments have to be true.

The capabilities of the tools we now have in order to truly prove such guesses are also becoming more powerful. Calculations and analytical and algebraic manipulations that could have taken many hours, and even years some time ago are now done in seconds. The possibility to hand over to different programs even the task of proving the conjectures at which we arrive through our experiments is a reality in some disciplines and soon will be expanded to many others.

For this reasons it is quite likely that in a rather near future a great part of the mathematical activity in research, teaching and learning, around any particular mathematical topic, will consist of the following tasks, in every step supported by the tools we now have at our disposal:

- i. to design with imagination, and guided by the wisdom gathered by us or by others, experiments that can direct us in the exploration of the subject we are considering
- ii. to try to make some initial guesses about the mathematical relationships that lie perhaps hidden under our experiments, our calculations, our graphical images, under the structures of different types we can discover
- iii. to confirm or to refute our conjectures, and to perfect them through more refined experiments, now more accurately guided through the previous explorations
- iv. to try to prove our final conjectures, perhaps by means of some type of automatic method

*(Sat July 13, 9:00, HS Ph)*

Albert D [Rich](#) (Hawaii, USA), [adr@flex.com](mailto: adr@flex.com)

### Automating the Simplification of Mathematical Expressions

Just about everyone who has used a computer algebra system to teach or do mathematics has, on occasion, been surprised and frustrated at the "simplified" results returned by the system. For this reason it is important to have a basic understanding of what is "under-the-hood" of the system you are using. Such knowledge makes it easier to obtain the results you want, and to understand those you don't.

As one of the principal authors of the Derive® computer algebra system and its predecessor, muMATH?, I would like to pass on some of the insights I have gained over the last 25 years implementing these systems. I will discuss the evolution of the principles and methodologies Derive uses to automate the simplification of mathematical expressions. Among the topics covered will be the definition and universality of simplification; the special difficulties presented by the calculus functions,

infinity, and its inverse; the three phases of the simplification process (parsing, expansion, and reduction); the relative merits of top-down vs. bottom-up simplification; and that most elusive goal: the commutivity of simplification and substitution.

Computer algebra systems are powerful tools, but they are no substitute for understanding the mathematics underlying the problem you are trying to solve. In short, if you are using Derive, or any computer algebra system, results should be viewed with a healthy skepticism and verified if at all possible.

*(Thu July 11, 9:00, HS Ph)*

Hans-Georg Weigand (University of Würzburg, Germany), weigand@mathematik.uni-wuerzburg.de

#### New Ways of Communicating via the Internet - New Ways of Learning

New technologies are rapidly changing our ways of communication and the art of teaching as well as extending ways of learning. Communication is an essential part of mathematics education and mathematics teacher education. Communications through technology can support the goals of mathematics education courses and programs, and can promote the professional development of participating teachers and educators. This talk is about:

My experience about Internet-supported lessons for mathematics teacher students; The development of Internet-based materials which can be used beside lessons and seminars in mathematics education and mathematics teacher education; The creation of a "virtual private network" between the university and a punch of schools and the intended collaboration between teachers and researchers.

*(Fri July 12, 9:00, HS Ph)*

## Lectures

Jay Abramson, Sharon Walker (Arizona State University, USA), abramson@math.la.asu.edu

#### Virtually Face-to-Face Mathematics Courses

For the last five years, Arizona State University has been actively involved in the research and development of online learning environments. During that time, we have come to understand how technology can support and enhance the teaching and learning process. Accordingly, we have developed two on-line courses, MAT 117, College Algebra and MAT 210, Brief Calculus. We believe quality instruction is not compromised when classes are conducted virtually, rather than in a face-to-face classroom. Through the use of advanced and emerging technologies, the same high educational standards can be adapted to anytime/anywhere learning environments. In these courses, the learning is dynamic and interactive. The courses are delivered on Blackboard Course structure, design, proper and appropriate pedagogy, interactive tips and student results will be discussed.

*(Wed July 10, 15:30, HS Ph)*

Bengt Ahlander (Ostrabo gymnasiet, Uddevalla, Sweden), ba@ostrabo.uddevalla.se

#### Let the students explore algebra with CAS, TI89

These symbolic calculating tools, CAS, Computer Algebra System, will change the way to teach mathematics more than the start of using graphing calculators did. With these tools we will get more time over to discuss the concepts of mathematics, more time to let the students explore algebra themselves and more time to increase the understanding of mathematics. The question is not if, but when and how, we should use CAS in our math classes. In my presentation I will show some examples how to work with TI89 and simultaneously reinforce the concepts of mathematics. All students in my class use TI89 and the age of the students are 17-19 years old.

*(Fri July 12, 12:00, HS 4)*

Giovannina Albano, Matteo Desiderio (Dip. Ingegneria dell'Informazione e Matematica Applicata, Italy), albano@diima.unisa.it

#### Improvements in teaching and learning using CAS

In this work we wish to present an innovative proposal for a differential equations course supported by software tools. The aims we have placed in such experience are principally three:

- we are interested in showing the students of the concrete applications of the daily life that can be described and interpreted through the use of the differential equations, discovering in such a way, the real adhesion between experience and mathematical environment, giving motivation to the students for the study of the subject;
- we want to add the interactive use of mathematical software to the traditional used didactic methodologies: in fact studies recently performed have put in evidence that the interactive use of such tools allows to arrive at a superior abstraction level in the solution of mathematical problems, avoiding more easily the happen of misconception due to the creation of wrong mental models;
- we want to stimulate the student to have a more critical attitude towards the solution of the problem described by a differential equation. It is well known in fact that the differential equations represent a base tool in the mathematical modelling in varied applicative fields as physic, astronomy, economic sciences and others. According to the problem which has determined the equation and to the kind of differential equation which is wanted to solve, we wish that the student to establish in critical way if it is necessary to find a solution qualitative or quantitative, in the second case if it must be

precise or approximate, and in this last hypothesis he must establish the wanted approximation level, to be able, on the basis of this, to choose an opportune solving method. In fact, a preliminary study of the equation can suggest the most suitable numerical method and vice versa an experimental study by numerical methods can provide information about qualitative appearances of the solution.

*(Thu July 11, 11:00, HS 4)*

Valeriu Anisiu (University Babes-Bolyai Cluj, Romania), anisiu@math.ubbcluj.ro

**Solving linear systems containing parameters with DERIVE**

In any Computer Algebra System, a linear system of equations containing parameters is solved only generically. For example, for a simple system as  $2x+3y=5$ ,  $4x+ay=10$ , only the solution  $x=5/2$ ,  $y=0$  is obtained, even if for  $a=6$  the system has an infinity of solutions. The situation in this example is however simple, because the determinant equals  $2a-12$  and the parameter value  $a=6$  can be examined separately. But for a null determinant or in the case of non square systems, the "singular" cases are harder to find.

The purpose of this paper is to obtain a general DERIVE function which is able to deal with such systems. The main idea is to use a customized version of the function ROW\_REDUCE which returns the Gauss-Jordan form of a matrix and the singular values of the parameters. The paper also contains the DERIVE programs and examples.

*(Fri July 12, 16:30, HS Ph)*

Klaus Aspetsberger, Brigitta Aspetsberger (Linz, Austria), aspetsberger@aon.at

**Data Collection and Mathematical Reasoning: An Attempt of Cross Curriculum Teaching in Natural Science Courses**

Cross curriculum reasoning is a principal objective of natural science courses in Austrian high schools. However in reality the students have problems to understand laws from physics and chemistry written in mathematical terms and to apply them to real situations. In math courses students often do not see the necessity for introducing new mathematical concepts. Examples demonstrating the use of these new concepts are often quite artificial and the students have even problems to understand these examples. Carrying out experiments in science courses the students have the possibility to produce their own data and to analyse them by mathematical reasoning. However, they have to learn experimenting, how to obtain good results, how to document their work and to write reports and how to work in groups.

CBL from Texas Instruments is a Calculator Based Laboratory which allows to collect data during physical and chemical experiments. Data are stored directly to a calculator e.g. the TI-92 for graphical visualisation and further manipulation. CBR from Texas Instruments is a motion detector which allows to gather a large amount of data points from an object in motion. CBL, CBR and TI-92 support data collection and manipulation. However careful experimenting is absolutely important for obtaining good quantitative results, which are necessary for functional modelling of experimental data. We report about experiments being carried out in the years 1999 to 2002. In several different classes consisting of students at the age of 16 to 18 experimenting with the CBL and TI-92 was integrated within regular classes. About 50% of the students were girls. A special course for high ability students at the age of 14 was installed during the school year 2000/01 also carrying out experiments with CBL. In 2000 a group of students were testing the water quality during regular classes using CBL and ion selective probes from Vernier.

We also report about experiences made during several in-service teacher training courses introducing CBL, CBR and TI-92 to math and science teachers within the last two years. It was surprising to see that they had problems similar to the students when treating experimental data.

*(Thu July 11, 10:00, Sem Ph)*

Angel Balderas (University of Querétaro, Mexico), balderas@sunserver.uaq.mx

**Integrating Derive in the didactics of Laplace Transform**

We present a didactic pathway of integration of CAS type software, specifically Derive, for the teaching and learning of the Laplace Transform in the context of differential equations courses at university level, working with 19 years old students.

Some phenomena linked with this integration are evidenced, such phenomena are related with teacher's preparation and with student's creativity. Some didactic problems due to the passage of doing math in a paper and pencil environment to doing math in information technology environment are discussed. We present the implementation with Derive of different processes associated with the topic: from calculating transforms to calculating inverse transforms, from the solution of differential equations to the solution of systems of differential equations, from the use of special functions to the use of the convolution theorem. Each one of the exposed ideas has been experienced in regular courses with students of different engineering careers.

*(Wed July 10, 15:00, HS 4)*

Yuriko Baldin (Universidade Federal de São Carlos, Brazil), yuriko@dm.ufscar.br

**Analysing the limitations of technology in teacher preparation courses**

Recently, many nice examples of technology-based classrooms and activities can be seen in the literature. Most of them are successful experiments or descriptions of activities with some specific software. With the advances on the research on new teaching methodologies, a new generation of mathematics teachers, either prospective or in-service, should be trained to use efficiently the information already available from different sources. In fact, one of the objectives of modern teacher

preparation courses is to prepare teachers with skills to analyze the existing material as well as to create their own activities. In this context, we raise some questions to be put into discussion:

- a) Are the teachers being prepared to realize the limitation of technology in teaching activities and to analyze the possible causes?
- b) Are the teachers being prepared to recognize any conceptual troubles in technology-based activities and to propose solutions?

The aim of this presentation is to analyze these questions through a comparative study of the capabilities of CAS, DGS and graphic calculators, based on concrete examples of mathematical content. We think that a discussion on these questions would improve the understanding of the role of technology in the educational context and its importance on the formation of teachers.

(Fri July 12, 11:00, HS Ph)

Linda Ball (Univ of Melbourne, Australia), lball@unimelb.edu.au

**So, what do I write down? I've solved the whole problem using CAS.**

In classrooms where students use CAS as their normal technology the approaches used to solve problems will most likely move from purely by-hand to a combination of by-CAS and by-hand. This will impact on the way students record their solutions to problems. How do students make decisions about what to record when most or all of their working is done using a CAS? This question requires a rethinking by teachers and students about the nature of good written records for solutions to problems. Students using CAS will still need to clearly communicate how they solved problems however their written solutions may look different to students who use pen and paper exclusively. Students who use CAS should not simply replace intermediate by-hand steps with calculator syntax. Instead they need to look more globally at how the written record communicates their solution. We suggest that students' written solutions should focus on the reasons for particular mathematical decisions, the overall plan of their solution, calculator inputs and selected answers from the CAS. This paper will report some initial work on student written records of solutions carried out as part of the CAS-CAT project. It will include a discussion of advice produced for teachers and students, teachers' comments about what they value in a written solution and sample student work.

(Fri July 12, 16:00 HS 1)

Rüdeger Baumann (Gymnasium Ernestinum Celle, Germany), baumann-celle@t-online.de

**Algorithmische Geometrie mit Derive**

An Beispielen aus der Analytischen Geometrie bzw. Vektorgeometrie wird gezeigt, wie sich dieses Gebiet der Oberstufenmathematik mit Hilfe eines Computeralgebra-Systems (etwa Derive) zu einer Algorithmischen Geometrie weiterentwickeln lässt. Die Schüler erstellen Konstruktionen in Gestalt von Algorithmen (bzw. Derive-Programmen) und beweisen deren Korrektheit. Es ist zu diskutieren, ob das (die Rechenmacht des) Computeralgebra-System(s) auch durchschnittlichen Schülern das selbständige Finden von Konstruktionen und Beweisen erleichtert oder allererst ermöglicht, wie diese sich von den herkömmlichen Beweisen und Konstruktionen unterscheiden und was vom "Geist der Geometrie" dabei noch übrig bleibt.

Die Teilnehmer sollen angeregt bzw. angeleitet werden, anhand von Aufgaben aus der Analytischen Geometrie / Vektorgeometrie bzw. Linearen Algebra eine (im Sinne des Vortrags) CAS-unterstützte Methodik des Konstruierens und Beweisens zu entwickeln.

(Wed July 10, 15:00, HS 1)

Michel Beaudin (École de technologie supérieure, Montréal, Canada), mbeaudin@seg.etsmtl.ca

**Using Both: Derive 5 and the TI-92 Plus**

The talk will give examples of how Derive 5 AND the TI-92 Plus can be used by the teacher and/or engineering students. It will compare some answers given by the two systems for various problems. Some of the examples will show that both systems can be used in partnership and this helps the students for a better understanding of basic concepts. This is what we have done for the last two years: using the TI-92 Plus with the students and switching to Derive when we think that it can do a better job. Because there is so many similarities between the 2 systems, using both systems is not too difficult for the students. Finally, based on our teaching experience, we will give some suggestions for future versions of Derive and the symbolic TI.

(Wed July 10, 10:00, HS Ph)

Lyudmyla Bilousova (Kharkiv State Pedagogical University, Ukraine), belousova@kgpu.sa.net.ua

**Assessment of Pupils' Knowledge in Mathematics**

Advantages of testing method for assessment of pupils' knowledge and the diagnostics of its structure are widely known: objective results, along with technological convenience and a lot of information obtained. Those are the reasons why both educators and pupils are interested in testing. We have developed some sets of tests on different topics in mathematics curriculum for secondary school. The following two kinds of tests have been developed:

1. Tests with tasks of different levels of difficulty. Those tests cover all parts and all levels of the subject topic chosen for testing.
2. Tests with answers of different levels. In this case each task corresponds to a particular element of the topic and the answers suggested for pupils' analysis reflect the increasing level of mastering the material.

Our tests have been used for thematic testing in Kharkiv secondary schools and enabled the teachers to realize the teaching function of testing. Also, those tests enhanced pupils' interest in the subject, made a solid basis for creating a system of individual monitoring for pupils' progress in learning, and, in addition, supplied us with some valuable information concerning the quality of teaching in Kharkiv schools.

*(Sat July 13, 12:30, HS 4)*

Sergey Biryukov (Moscow State Pedagogical University, Russia), svb3@hotmail.ru

**Dimensionless Products Derivation in DERIVE**

Dimensional analysis is very helpful as a complex natural science problems solving support. The use of dimensional products usually reduce the dimension of the problem to be solved and thus dramatically reduce the number and complexity of the real world and computer simulations. As the number of dimensional products sets are dozens for real problems and the derivation of each product need several high order determinants CAS DERIVE was used for dimensional matrix generation, base units powers calculation and clear mathematical form representation. The appropriate utility DIM\_PROD is written. Its main function DIM\_PROD() argument is a two column matrix of parameters names and its dimension. The result is a vector of sets of independent dimensional products expressed in parameters names. The utility supports TI-92+ - like units notation, default units customization, units conversion and equations reduction to the dimensionless form.

*(Fri July 12, 14:30, HS Ph)*

Sergey Biryukov, Dmitry Guskov, Igor Makarov (Moscow Pedagogical State University, Russia), svb3@hotmail.ru

**Deriving Real World Data**

The main aim of our current project is to make a CBL-like (CBL is a Calculator Based Laboratory by Texas Instruments Inc.) system using DERIVE and regular computer input/output ports (Game, LPT, COM) and such regular devices as mouse, joystick, sound card, WEB-camera etc. Three examples of laboratory works are presented: "Fresnel Equations", "Damped Oscillations", "Alternating Current Circuits". The first one uses DERIVE for DOS, original data collecting program (Pascal) and a keystroke simulator Scancode. The 2nd and the 3d uses DERIVE for Windows and original (Kylux for 2nd) and commercial (Sound card based oscilloscope for the third) data collection programs and Hotkey keystroke simulator.

*(Fri July 12, 12:00, HS 1)*

Sergey Biryukov, Valery Kuzminikh (Moscow Pedagogical State University, Russia), svb3@hotmail.ru

**The Role of Random Perturbations and Control Vector Restrictions in the Problem of Spacecraft Optimal Control**

The problem of the spacecraft trajectory optimization in the presence of random perturbations of the control vector is usually solved by the reduction to the boundary problem of Bellman equation. We extended the above problem with the restricted orientation of control vector - the system of linear equations. Bellman's equation is transformed to the quasilinear parabolic PDE and an approximate symbolic solution for the case of initial function that is quadratic in adjustment parameters is derived. CAS DERIVE was helpful in substitutions and symbolic manipulations, especially bulky 3D Poisson integral integration. The result is visualized in 3D with DERIVE and the role of perturbations and restricted direction of control vector is analyzed.

*(Wed July 10, 10:00, HS 3)*

Bill Blyth (Department of Mathematics, RMIT University, Melbourne, Australia), bill.blyth@rmit.edu.au

**Finite Element Methods: Presentation and Animation using Maple**

Maple is widely used in our courses. The role of Maple varies considerably from a small support activity to "immersion". We teach a Finite Element Methods course to a large number of engineering students in their third or fourth year. Most of this FEM course uses Maple to do all of the computation and students are provided with Maple files which they need to edit in order to complete the assignments. The final part of the courses uses commercial FEM software.

This paper discusses how Maple is used as the presentation medium (as well as the computational tool) in lectures for the Maple part of the course. This includes an animation of the error for a collocation method solution of a Boundary Value Problem (BVP) as the collocation points are changed. This animation cannot be achieved by a naive use of the animate command - the required methodology to produce a more complex animation is discussed.

*(Sat July 13, 11:30, HS 4)*

Denis Bouhineau, Jean-François Nicaud, Thomas Huquet (IMAG-Leibniz, Grenoble, France), Jean-Francois.Nicaud@imag.fr

**The Aplusix-Editor**

The Aplusix-Editor is a computer system (also called Edix in some papers) that aims at filling a gap, among systems for algebra, between tutors like MathXpert and PAT, and Computer Algebra Systems (CAS). Here is the gap: on one hand, tutors allow the student to solve problems by applying small local commands to sub-expressions (e.g., combine like terms, extract a common factor). In the best cases, the system can help the student to solve problems step by step and can give some explanation, but the system is problem-directed and limited to a particular domain defined by the tutor. On the other hand, CAS are general purpose systems for doing mathematics without any concern/insight for a particular domain-problem. CAS allow the student to solve problems in one step by applying strong global commands (e.g., factor a polynomial, solve an equation). Neither help, nor explanations are available. In both systems, the student cannot produce his/her own steps according to his/her own tempo and have them just verified by the system.

With the Aplusix-Editor, the student produces his/her own steps with the help of an editor of algebraic expressions. At each input, when the typed expression is well-formed, the system calculates the equivalence between the expression at the current step and the expression at the previous one, and tells the result to the student by drawing an equivalence link that is crossed in red when there is not equivalence. The editor is an advanced two-dimension editor that has better algebraic properties than the few current ones (e.g., MathType or the editor of Mathematica). The equivalence is currently calculated for a class of expressions including polynomials, systems of linear equations, equations and inequalities of one variable with a degree less or equal 3.

The Aplusix-Editor has been experimented with two groups of 9th grade students: a regular class and a group of volunteers in a controlled experiment out of the class. The results are very positive: the students learned some algebra and liked to use the system.

We are currently working to add features to the Aplusix-Editor to get an educational CAS. These features are small local commands like tutors' ones and strong global commands like CAS ones.

The presentation will include a demonstration of the system.

(Fri July 12, 14:00, HS 1)

Hans-Joachim Brenner (Albert-Schweitzer-Gymnasium, Erfurt, Germany), Hans-Joachim.Brenner@t-online.de

#### Diskussion der Ziele des Mathematikunterrichtes anhand von Unterrichtsbeispielen

Seit drei Jahren wird der TI-89 im Mathematikunterricht der Klassen 10 bis 12 unseres Gymnasiums genutzt. Erstmals ist in diesem Jahr ein CAS bei den Abiturprüfungen in Thüringen zugelassen (8 beteiligte Schulen). Im Vortrag möchte ich Unterrichtsbeispiele vorstellen und Vorschläge unterbreiten, wie auf der Grundlage bisher gültiger Lehrpläne der Unterricht gestaltet werden kann, wie ein Lehrplan den neuen Gegebenheiten angepasst werden sollte und welche Schwerpunktsetzung bei den Zielen des Mathematikunterrichtes aus meiner Sicht sinnvoll ist. (Die Bewältigung des Alltages muss stärker in den Mittelpunkt der Diskussion gestellt werden. Und dafür ist es notwendig, dass man sich auf grundlegende Ziele einigt.)

Im Einzelnen habe ich folgende Beispiele vorgesehen: a) Abstandsbestimmungen im Raum, b) Bilder rationaler Funktionen, c) Untersuchung rekursiver Zahlenfolgen. Dabei werde ich Möglichkeiten aufzeigen, wie im Unterricht die einzelnen Teilgebiete der Mathematik stärker als oft üblich miteinander verknüpft werden können und wie in Leistungskontrollen auch „einfache Punkte“ für den Schüler erreichbar sind.

(Fri July 12, 15:00, Sem Ph)

Tatyana Byelyavtseva (Kharkiv State Pedagogical University, Ukraine), byelyavtseva@kgpu.sa.net.ua

#### Teaching Divisibility of Numbers with Computer Technology

Divisibility of numbers is studied in different parts of secondary school algebra. Teachers notice that pupils usually experience difficulty in studying this topic. To help pupils to study this topic quickly and easily the program "Divisibility of Numbers" has been developed. This program can be used for two purposes: for learning and for testing. While pupils learn new material the program supply them with all the relevant information on divisibility of numbers. The program also gives pupils the opportunity to do some exercises on divisibility. When pupils test their knowledge the program gives them a number of tasks and correct (as well as records) their activity. The methods of using this program in classes on algebra has been developed. The program "Divisibility of Numbers" has been successfully used in secondary school. The results showed that the use of the program helped to increase pupils' understanding of the topic and made the process of learning much more enjoyable.

(Sat July 13, 12:00, HS 4)

Mary Ann Connors, Edward A Connors (Westfield State College, Easthampton, USA), connorseds@charter.net

#### A Predator-Prey Model With a Computer Algebra System

The purpose of this paper is to present a predator-prey model using a hand-held computer algebra system (TI-89/TI-92 Plus) to enrich and enhance the learning of ordinary differential equations and linear algebra. The example provides the opportunity for student active learning. This practical application of a second-order model will be examined from graphical, numerical and analytical points of view.

Students can represent differential equations and their solutions in various geometric ways. For example, students can readily use time graphs or solution curves graphed as a time series plot and/or a phase portrait to better understand long term behavior. Tables also provide information. Matrices, eigenvalues and eigenvectors are also easily handled on the TI-89 to determine the exact solutions to systems of ordinary differential equations. In this presentation we will demonstrate how students use the technology to analyze the predator prey model from biology. We will investigate time and phase portrait plots related to the model.

(Fri July 12, 10:00, HS 4)

Eliane Cousquer, Pierre André Caron, Anne Midenet (Univ. of Sciences and Techn. of Lille, France), Eliane.Cousquer@univ-lille1.fr

#### Initial Training of Teachers of Mathematics and collaborative work in the IUFM (Workshop)

For several years, Technologies of Communications have been intensively used by the trainers in mathematics of the training college I.U.F.M. of the North of France. Use of the e-mail for the memoirs of maths, use of WEB resources for the history of mathematics, an on-line bibliographic data base, a data-base of memoirs and web sites created by trainers: the C.R.E.A.M, a center of pedagogical resources for the trainees; Mathadoc (A6-3), the electronic schoolbag of the secondary school teacher

that the teacher can modify it the way he wants.; LILIMATH : discovery workshops for a use in the classes received in 1998 the first prize of a national competition (cervod) of software tools for training; FUNCTIONS allows an individual following-up of the works of the pupils (fourth price in that same competition) ; GEOWEB, a site presenting some creations made by the pupils themselves on the solving of some open problems of geometry, has received in 2001 a national price for innovative methods. All these groups of teachers use e-learning software for their network.

In 2001-2002, a new program for the training in technologies has begun, in link with the memoirs. The trainers are themselves engaged in creation of ressources and animation of networks, and a workshop with the trainers engaged in the memoirs will analyze this new training. This program is part of a program called FORMASCIENCES, with laboratories of the North of France; the essential points of this program are the following: use of simulations in sciences, real teaching program, with references to distributed cognition and collaborative work. This communication will give the essential point of this program and a paper will analyze the first year of training.

*(Thu July 11, 12:00, HS 3)*

Patricia Cretchley (University of Southern Queensland, Toowoomba, Australia), cretchle@usq.edu.au

**Mathematics and Technology: how integrated is this learning partnership?**

Recent research by the author and others has revealed remarkably weak correlations between early undergraduate students' confidence in their ability to do and learn mathematics, and their confidence in using computers, in a range of technology-enriched mathematics learning programs. Clearly this phenomenon has implications for educators who seek to integrate technology effectively into the learning of mathematics. This paper reports on a pilot investigation of the influences that these and related attitudes may have, when students face tasks commonly encountered in early undergraduate mathematics. The study used well-established scales to establish the range of confidence levels for 176 students in a typical Australian first-year undergraduate mathematics Linear Algebra and Calculus course. A representative focus group of 30 students was identified for observation late in the semester, and their attempts (handwork and technology) at eight specific tasks were captured in their usual computer laboratory sessions, where they could use a computer or graphics calculator. The tasks were kept simple, constructed so that the available technology was of varying degrees of use, and required techniques that encapsulated the essence of the much wider range encountered over the semester: solving systems of linear equations, finding determinants, and calculating definite integrals. Students' responses were assessed to establish two outcomes: how appropriate their choices of approach were, and the degree to which they were empowered by the use of technology. Levels quantifying these "approach" and "empowerment" outcomes were established for each student, and are illustrated on a scatter-plot. The plot distinguishes between four distinct quadrants of students: those who are confident in both the learning of mathematics and their ability to use computers, those who are not confident in either, and those who are confident in one of the two areas, but not the other. Their responses to technology are compared, implications are considered, and possible directions for further research are discussed. An Appendix gives the scales used to measure mathematics confidence, confidence in the use of computers, and attitudes to technology in the learning of mathematics, with Cronbach alpha reliability ratings.

*(Thu July 11, 12:00, HS 1)*

Bernard Cunningham (Mott Community College, Vassar, Michigan, USA), bcunning@gfn.org

**Symbolic Math Guide - Two Years of Data on an Introduction to Integrated Math Course**

In the winter & spring of 2001, the students at Lakeville High School in Otisville, Michigan, USA were involved in the piloting of the SMG Application in two ways. A Precalculus class helped evaluate the application before it became public with a very critical eye. The second phase was the involvement of 90 students who would not be classified as model students. These 90 students were enrolled in a class entitled "Introduction to Integrated Mathematics". A pre-test was given. The students were then introduced to the TI-92 calculator and the SMG Application. After some time a post test was given (which was identical to the pre-test) The data is very interesting.

The second set of data will be from the 2001-02 school year with 50 students from the same titled class. At the conclusion of presentation of the data from both years the floor will be open for questions and discussion of the Symbolic Math Guide Application.

*(Thu July 11, 12:00, HS 4)*

Robert Decker (University of Hartford, Connecticut, USA), rdecker@mail.hartford.edu

**Building Interactive Mathematics Software**

Computer algebra systems are not strong at "fast feedback" exploration. In order to see how a parameter change affects the graph of a function or differential equation one must edit and then reissue a command, which is slow and inconvenient.

Tcl/Tk is a free platform independent (mac, windows, linux, unix) language that can be used to quickly build interactive mathematics programs. I have created a number of helper functions (components) which make it even easier for a mathematics educator to create little programs to illustrate particular concepts. These interactive programs can be used as stand-alone programs or as front-ends to computer algebra systems (explore and then transfer the work to the computer algebra system).

I will demonstrate some interactive graphing programs that illustrate parameter changes for functions and differential equations, changes in initial conditions for differential equations (it becomes very easy to generate a complete phase portrait when done interactively), and interactive data fitting. I will also discuss benefits to both teachers and students with this

approach, and some possible research directions in mathematics education related to this "write and then use" interactive software approach.

*(Sat July 13, 10:00, HS 4)*

Eva Dimitrova, Penka Georgieva (Higher Institute of Food and Flavour Industries, Plovdiv, Bulgaria), [eva\\_di@hotmail.com](mailto:eva_di@hotmail.com)  
Research on the influence of some factors on the teaching of Mathematics in The Higher Institute of Food and Flavour Industries

We investigated the influence of the following factors: type of secondary school the students have come from, the entry test type and the major. Our research covers several student groups formed on the basis of the factors chosen. There is a clearly formed tendency for the students from secondary technical schools to show a better ability to cope with the mathematical courses in our Institute and to have higher ratings than those from high schools. There are also differences in adaptation and in ratings between students in Technical and Technological Faculties.

Our study and the conclusions we have drawn enable us to analyze the situation and to prepare more precise and detailed approach to the different groups in developing innovative methods in teaching Mathematics using educational software such as the mathematical assistant CAS DERIVE.

*(Wed July 10, 12:30, HS 4)*

Hans-Jürgen Elschenbroich (Studienseminar S II Neuss, Germany), [elschenbroich@t-online.de](mailto:elschenbroich@t-online.de)

Dynamisch Funktionen entdecken

Funktionen sind ein wichtiges Themengebiet in der Sekundarstufe I. Üblicherweise werden sie als Teil der Algebra unterrichtet und neue Technologien werden höchstens genutzt, um die Graphen zu plotten. Auf diese Weise bekommen die Schüler eher ein statisches Bild von Funktionen. Mit Dynamischer Geometrie-Software (hier Euklid-Dynageo) ist jedoch ein dynamischer Zugang möglich! Die Schüler können zum einen die unabhängige Variable  $x$  variieren und untersuchen, wie verändert sich  $y$  und wie entsteht der Funktionsgraph als Ortslinie von  $P(x/y)$ . Sie können weiter die Parameter  $a$ ,  $b$ ,  $c$  variieren und beobachten, welche Auswirkungen das für den gesamten Graphen hat. Dies lässt sich einfach mit Schiebereglern durchführen, die Auswirkungen sind unmittelbar zu sehen. Auf diese Weise erhalten die Schüler ein neues Werkzeug, um Eigenschaften von Funktionen zu entdecken. Das soll am Beispiel quadratischer Funktionen exemplarisch gezeigt werden.

*(Fri July 12, 15:30, HS 4)*

Hans-Jürgen Elschenbroich (Studienseminar S II Neuss, Germany), [elschenbroich@t-online.de](mailto:elschenbroich@t-online.de)

Dem Höhenschnittpunkt auf der Spur

Mit Hilfe Dynamischer Geometrie-Software (hier Cabri II) kann nicht nur zu einem einzelnen Dreieck der Höhenschnittpunkt konstruiert werden, sondern es kann seine Ortslinie untersucht werden, wenn die Gestalt des Dreiecks verändert wird. Wird die Ecke  $C$  auf einer Parallelen zu  $AB$  bewegt, ergibt sich eine Parabel. Der Nachweis, dass so die erzeugte Linie tatsächlich eine Parabel ist, erfolgt über klassische Abstandsüberlegungen. Derartige Aufgaben waren über Jahrzehnte aus dem Unterricht verschwunden, sind aber noch in Schulbüchern aus den 50er Jahren zu finden! Es wird gezeigt, wie diese klassischen Ansätze mit Dynamischer Geometrie-Software wiederbelebt werden können. Anschließend werden die Ergebnisse auf quadratische Funktionen in der SI-üblichen Funktionsschreibweise angewandt.

So werden ausgehend von einem elementaren Problem Querverbindungen von Algebra und Geometrie mit Hilfe von DGS (wieder) aufgebaut, Aspekte, die aus unserem Mathematikunterricht fast verschwunden sind.

*(Fri July 12, 12:00, Sem Ph)*

Johann Engelbrecht, Ansie Harding (University of Pretoria, South Africa), [jengelbr@scientia.up.ac.za](mailto:jengelbr@scientia.up.ac.za)

Internet Calculus: How does Big Brother watch?

The instructor stays mainly in the background when presenting an internet course and does not have the hands on opportunity to motivate students and check daily progress. Yet, it is still largely the instructor's responsibility to present a successful course and to help maximise the opportunity of every student to succeed. It is therefore important to monitor the progress of students and act when required. One possible tool for doing this is to employ co-operative learning where students are divided into small groups and are responsible for assignments and projects as a group, thus letting the group synergy act as a catalyst for performance. Another tool that can be used for letting the student monitor his or her own progress is that of internet quizzes. We discuss our experience in using these two tools. We investigate the benefits and difficulties and the overall success.

*(Wed July 10, 11:00, HS 1)*

Terence Etchells (Liverpool John Moores University, UK), [t.a.etchells@livjm.ac.uk](mailto:t.a.etchells@livjm.ac.uk)

Z Transforms, Derive and High Order Linear Recurrence (Difference) Equations

With the advent of the new programming constructs in Derive 5, this lecture describes an example of how Derive's mathematical capabilities can be significantly extended using these constructs. In particular we show how the string function in Derive 5.04+ can be used to determine the structure of mathematical expressions entered into user defined functions.

The Z transform and its inverse are not internal functions in Derive, but we describe Derive 5 functions that will produce both Z and inverse Z transforms for certain classes of functions. We define and program known results for calculating Z

transforms and apply complex analysis to automate the calculation of the inverse Z transform. Further we apply the Z and inverse Z transform functions to the solution of high order linear recurrence (difference) equations, significantly extending Derive's current ability to solve only first and second linear difference equations. We conclude by highlighting a few improvements that would greatly help the Derive programmer construct, edit and debug their programmes.

*(Thu July 11, 11:00, HS 1)*

Peter Flynn (Department of Science and Maths Education, The University of Melbourne, Australia),  
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**Technical and Pedagogical Challenges in Constructing CAS-Permitted Examination Questions: Some Principles**

The advent of CAS-permitted mathematics subjects in upper-secondary school systems present many technical and pedagogical challenges to examination authorities and textbook authors. A study of questions from non-CAS examinations suggests that CAS renders some traditional questions redundant for technical and/or pedagogical reasons and for others, it shifts the balance of abilities assessed. In a technical sense, CAS influences many facets of examination question design such as structure, language, explicitness, accessibility to users of different CAS and marking schemes. On a pedagogical level, careful consideration needs to be given to the nature and bandwidth of mathematical abilities we want (and are able) to assess in CAS-permitted examinations. One mathematical activity requiring careful scrutiny is the assessment of proof especially when the construction of the proof is achieved mainly by symbolic manipulation and/or use of algebraic or trigonometric identities. Many non-CAS examinations contain question parts structured around instructions such as 'show that' or 'prove' and these are likely to be affected by CAS. In this paper, the suitability for the future use of 'show that/prove' questions is scrutinised from this dual perspective and reasons for any lack of suitability are highlighted. As CAS can alter the number and nature of required intermediate steps to such questions, a set of principles for the language and structure of 'proof' questions will be proposed and discussed using international examples and data from The University of Melbourne's CAS-CAT project.

*(Wed Jul 10, 12:00, HS 1)*

Hongguang Fu, Zhenbing Zeng (Chengdu P.R. China), Fu\_hongguang@hotmail.com

**A New Dynamic Geometry Software with a Prover and a Solver**

As we know, dynamic geometry technology has been used in mathematics education widely, and many dynamic geometry software such as GSP and Cinderella etc has been developed. But, all these software do not have the function to solve problem by computer automatically. In order to do it, we must introduce a automated reasoning engine (prover) and a symbolic computation platform (solver). Moreover, for the goal of education, the prover and solver should produce a readable process similar to it given by hand. In recent years, we are researching and developing such a new dynamic software with a prover and a solver. By the end of last year, the project almost finished. So in this paper we will illustrate our software and show some interesting examples. We believe that the new dynamic geometry software will very helpful for students and teachers.

*(Fri July 12, 14:00, HS 4)*

Györgyi Führer-Nagy (Westungarische Universitaet, Sopron, Hungary), fuh@emk.nyme.hu

**Ausrechnen Einer Regressionskurve mit TI-83**

In Kreisen der Jäger ist es bekannt, dass die Schnecke von des Muffelwildes staendig, im höheren Alter aber langsamer, wächst und deren Länge sich einem Maximumwert A nähert. (Verschiedene Jagdgebiete haben verschiedene Lebensbedingungen, und damit ist auch Qualität der Jagdtrophäen verschieden.) Die Veränderung der Länge der Schnecke versuchen wir mit einer Kurve modellieren. Die Gleichung dieser Kurve ist:  $y = A(1 - abx)$ , wobei x das Alter (Jahre), y die Länge (cm) und A die theoretisch maximale Laenge (für das untersuchte Land) zeigen. In Wirklichkeit ist es eine in die exponentiale veränderbare Regression, einfach mit  $y^* = abx = 1 - y/A$ . klassische Bestimmung dieser Regression ist ziemlich mühsam. Wenn uns ein Taschenrechner TI-83 zur Verfügung steht, wird alles deutlich einfacher. Programm für Ausrechnen der Tabellenwerten und Koeffizienten der Exponentialkurve ist schon in Grundoperationen gegeben.

*(Thu July 11, 12:00, Sem Ph)*

Igor Gachkov (Karlstad University, Sweden), Igor.Gachkov@kau.se

**Discrete Mathematics: Boolean algebra and Set theory with TI-89**

The packages "Boolean algebra" and "Set theory" are a program packages, which was created especially for TI-89 and is used for teaching in course of Discrete Mathematics based on a traditional textbook Ralph P. Grimaldi, Discrete and Combinatorial Mathematics an Applied Introduction, fourth edition. Actually this package is a natural development and a further edition of the package "Boolean.m" in MATHEMATICA, which has been used by the author during a long time for teaching in this course. The packages "Boolean algebra" and "Set theory" for TI 89 contains programs which allow to display different steps with illustrative explanations calculation in the Boolean algebra and Set theory.

Calculators allow to change the teaching process by replacing of MATHEMATICA with TI-89 due to their safety, low price and because they are easy to use and are possible to develop and provide. Of course MATHEMATICA has more powerful calculating possibilities, but calculators are very flexible, and can therefore be used during lectures in big rooms without technical facilities.

*(Fri July 12, 12:30, HS 4)*

Jose Luis Galán García, Pedro Rodríguez, Ángeles Galán, Yolanda Padilla (University of Málaga, Spain), [jl\\_galan@uma.es](mailto:jl_galan@uma.es)

**COMPLEX.MTH: Solving Problems of Functions of a Complex Variable for Engineering Using Derive**

In this paper we present the file *COMPLEX.MTH*, created for being used in subjects that deal with complex variable, aimed at Engineering students. Such file contains a series of macros which permit to solve some typical problems related to complex variable. The macros contained in the file can be grouped within the following blocks:

- Complex numbers (different forms of a complex number, arithmetic of complex numbers, loci in the complex plane).
- Functions of a complex variable (real and imaginary parts of a complex function, standard functions).
- Differentiation of complex functions (Cauchy-Riemann conditions, analytic and harmonic functions).
- Complex integrals (line integral of a complex function, analytic function integration, Cauchy's integral formula, the residue theorem).
- Conformal mapping (basic conformal mapping, fundamental theorem).

We also show in the paper some examples of applications that have been carried out with our students of Telecommunication Engineering. Finally, we include the conclusions obtained after using this file.

*(Wed July 10, 15:30, HS 4)*

Jose Luis Galán García, Pedro Rodríguez, Ángeles Galán, Yolanda Padilla (University of Málaga, Spain), [jl\\_galan@uma.es](mailto:jl_galan@uma.es)

**ANALVEC.MTH: Integration and Vector Field Problems for Engineering Using Derive**

In this paper we present the file *ANALVEC.MTH*, created for being used in subjects that deal with integration and vector field theory, aimed at Engineering students. Such file contains a series of macros which permit to solve some typical problems related to vector fields. The macros contained in the file can be grouped within the following blocks:

- Scalar and vector fields (gamma and beta functions, gradient, divergence, curl and laplacian).
- Line integrals (exact differentials, potential function, line integral of non-exact differentials).
- Double and triple integrals (double integration in cartesian and polar coordinates, triple integration in cartesian, cylindrical and spherical coordinates).
- Surface integrals. Gauss' theorem (surface area, unit normal vectors, flux of a vector field, Gauss' theorem).

We also show in the paper some examples of applications that have been carried out with our students of Telecommunication Engineering. Finally, we include the conclusions obtained after using this file.

*(Wed July 10, 13:00, HS 4)*

José Luis Galán García, Pedro Rodríguez, Yolanda Padilla, Ángeles Galán (University of Málaga, Spain), [jl\\_galan@uma.es](mailto:jl_galan@uma.es)

**Teaching Mathematics in Engineering with Derive and Experience in the University of Malaga**

In this paper we present an experience carried out in courses for undergraduate students in Technical Telecommunication Engineering. The normal use given to practicals with computers generally consists of solving specific problems using the primitive functions of the software chosen. The experience we present consists of teaching Mathematics with DERIVE using some kind of innovative practicals in order to use the computer as a tool for mathematical creativity. The main innovative aspect of these practicals is that the students have an active role in the sense they should elaborate themselves utility files to solve the typical problems for the different subjects. This fact implies that the students need to deal with programming in DERIVE, understand the subject and know how to solve typical problems. We will explain the reasons for choosing DERIVE and the details of developing these practicals. We will focus on a specific practical. We will end with the obtained conclusions and with the corresponding references.

*(Thu July 11, 10:00, HS 4)*

Tena Golding (Southeastern Louisiana University, Hammond, LA, USA), [tgolding@selu.edu](mailto:tgolding@selu.edu)

**Assessing Geometric Concepts with the Digital Camera**

The proposed session will discuss a digital camera project used in a geometry class for future elementary school teachers. The project, "Geometry is All Around Us," was designed to help assess students' ability to connect the course material to their everyday surroundings. This end-of-semester group project required at least 10 different photographs reflecting 10 different geometric concepts that had been covered in class. The geometric properties of each photo had to be stated in the form of a caption, story, etc. When the projects were shared with the rest of the class, rich discussions not only helped reinforce some concepts and help clarify possible misconceptions but also served as a nice review for the final exam. Since the format of the final project was left up to the students, the various submissions included posters, scavenger hunts, storybooks, I-spy games, etc. which provided a variety of very creative presentations. During the proposed session sample projects will be shared as well as the rubric for the evaluation of the project. Students' reactions to the project will also be discussed.

*(Sat July 13, 11:00, HS 4)*

Sonnhard Graubner (Wittgensdorf, Germany), [Sonnmath@aol.com](mailto:Sonnmath@aol.com)

**Using Computer Algebra in triangle Geometry**

We demonstrate how computer algebraic methods can be employed to solve problems from classical geometry. An inner point of a plane triangle is called a Brocard point iff the straight lines connecting it with the vertices form three equal angles with the adjacent sides. Any plane triangle contains two Brocard points. We investigate whether there are triangles for which both

Brocard points are situated on the incircle line. By an analytical ansatz we obtain a system of algebraic equations which are subsequently solved using the Groebner package included in the Maple system.

The main result is that there exist (up to similarity) exactly four triangles with the required property. Two of them are isocoles and constructible by compass and ruler. The approach allows to attack a wide class of geometrical incidence problems which could not be handled manually due to their algebraic complexity.

*(Wed July 10, 12:00, HS 4)*

Matthias Goldgruber (Institut für Softwaretechnologie, TU Graz, Austria), goldy@sbox.tugraz.at

**Eine explizite Hierarchie von Typen elementarer Gleichungen**

Algebra Systeme erweisen sich als höchst leistungsfähig im Zuordnen einer Gleichung zur passenden Lösungsmethode -- es ist eine vergebene Lerngelegenheit, dass sie dies automatisch und vor dem Benutzer verborgen tun.

Das ISAC-Projekt zielt auf ein Mathematiksystem, das seine Schritte in Interaktion mit dem Benutzer durchführt. Im Fall des Gleichungslösens sind dies u.a. die explizite Spezifikation des Gleichungstyps, sowie die Auswahl der zur Lösung der Gleichung vorgesehene Methode. Der Vortrag stellt das Konzept der in ISAC entworfenen Hierarchie von allgemeinen Problemtypen am Beispiel von Gleichungstypen vor, erklärt kurz die zur Implementation verwendeten grundlegenden Techniken aus 'Symbolic Computation' (matching, rewriting, term orders), und demonstriert die in ISAC verfügbare neuartige Funktionalität interaktiven Spezifizierens mit Benutzerführung. Die Beschreibung der Problemtypen erfolgt in ISAC mit Hilfe der üblichen Formelsprache und getrennt vom restlichen System; deshalb kann die Hierarchie mit beliebigen Problemen aus speziellen Unterrichtseinheiten erweitert werden.

*(Fri July 12, 16:30, Sem Ph)*

Andreas Griesmayer (TU-Graz, Institut fuer Softwaretechnologie, Austria), agriesma@ist.tu-graz.ac.at

**Tools fuer die interaktive Spezifikation von Problemen**

Seit der Einfuehrung von algebraischen Systemen (CAS) in die mathematische Ausbildung hat sich die Bedeutung der Spezifikationsphase erhoeht. CAS-Tools bieten eine gute Rechnerunterstuetzung, stellen aber keine Hilfe bei der Spezifikation zur Verfügung: Auswaehlen einer Funktion mit den passenden Parametern muss nach der Spezifikation gemacht werden. Das Ziel des ISAC-Projekts ist ein "educational CAS", das alle Phasen unterstuetzt: modellieren, spezifizieren und loesen. Zum Zweck der interaktiven Spezifikation wird in ISAC das mathematische Wissen in einem dreidimensionalen Universum entlang der Axen "domains", "problem types" und "methods" verwaltet. Der Vortrag behandelt das Problem bei einer so grossen und komplizierten Wissensbasis die UEbersicht zu behalten sowie entsprechende Tools um dies zu bewerkstelligen. Es werden bereits implementierte Tools zum Durchsuchen der Problemtyp-Achse und dem interaktiven Spezifizieren vorgestellt. Weiters werden neue ISAC-features zum Spezifizieren und Loesen von Problemen durch Aufteilung in Teilprobleme praesentiert.

*(Wed July 10, 13:30, Sem Ph)*

Gary Harris (Texas Tech University, USA), harris@math.ttu.edu

**Distance Delivery of a Graduate Level Mathematics Course for High School Teachers**

Here we report on the development and implementation of a graduate level mathematics course taught via the internet for in-service high school mathematics teachers. The course uses WebCT, MAPLE, and MWS Logo. We discuss the philosophy and rationale for the development of the course, the goals of the course, and the logistics involved with the distance delivery of the course. Also we provide examples of the course materials. We include profiles of the students and instructor involved with the course, and conclude with the assessment of the course's success from the point of view of both the students and the instructor.

*(Wed July 10, 11:30, HS 4)*

Judith Hector (Walters State Community College, Morristown, TN, USA), Judy.Hector@ws.edu

**Problem Solving, Programming, and Pedagogy**

Good problem solvers are in short supply for careers in science, mathematics, engineering and technology (SMET). This paper reports on how teaching programming on TI-89/92 calculators enhances student problem solving skills and mathematical understanding. The author has taught computer programming for SMET students at an American high school and a two-years college (first two years of university.) She uses a high level computer language (FORTRAN) on a main frame in parallel with TI-89/92 language. The structured programming topics include top-down design, step-wise refinement and algorithm development. The paper discusses pedagogical issues and suggests a sequence of instruction to develop understanding of uses of iteration and recursion. In learning programming, students read, flowchart and modify already developed programs before solving problems with programs they develop themselves. CAS software offers preprogrammed approaches to problems such as root finding and numerical differentiation and integration. One reason for teaching programming is so that a student will be able to apply such software to non-routine problems.

*(Wed July 10, 12:30, HS Ph)*

Rainer Heinrich (Pestalozzi-Gymnasium Dresden, Germany), Raihein@aol.com

**Ein Fach verändert sich - Erfahrungen mit der verpflichtenden Einführung von Grafikrechnern in Sachsen.**

Im "Zentralabiturland" Sachsen (Bundesrepublik Deutschland) wurde im Jahr 1996 für alle Schüler ab Klasse 8 die Nutzung des grafikfähigen Taschenrechners verbindlich vorgeschrieben. Im Vortrag sollen Erfahrungen und Auswirkungen dieses Schrittes auf den Mathematikunterricht gezeigt werden. Diese betreffen Inhalte, Aufgabekultur und didaktische Gestaltung des Unterrichts, Konsequenzen für die Bewertung und Zensierung und auf Schüler- und Lehrertätigkeiten. Insbesondere soll die Positionierung zum Basiswissen für Mathematik, die sich nicht an traditionellen Linienführungen von Lehrplänen orientiert, vorgestellt werden.

(Wed July 10, 15:00, Sem Ph)

Thomas Himmelbauer (Gymnasium Neulandschule Wien, Austria), j.himmelbauer@chello.at

**Aufgaben aus der Bewegungslehre mit dem TI-92 Plus**

An Hand mehreren Beispielen soll demonstriert werden, wie mit Hilfe der Parameterdarstellung von Funktionen des TI-92 Plus Bewegungslehre veranschaulicht und in den Mathematikunterricht als Anwendung integriert werden kann. Die Behandlung des freien Falles, des schiefen Wurfs, von Brems- und Überholvorgängen von Kraftfahrzeugen, des Wendekreises von Fahrzeugen, der Zentripetalbeschleunigung von Fahrzeugen in Kurven und von Verfolgungsproblemen benötigt Vektorrechnung, genauso wie Trigonometrie oder Analysis. Dabei hilft der TI-92 bei der Veranschaulichung, entlastet bei der Berechnung und ermöglicht so die Bearbeitung für den Schüler schwierigerer, aber auch interessanter Aufgaben.

(Sat July 13, 11:00, Sem Ph)

Fernando Hitt, Manuel Santos (Universite de Quebec a Montreal, Canada), fhitta@data.net.mx, msantos@purdue.edu

**Searching for Advantages of the Technological Dimension in Mathematical Problem Solving**

Summary: Teachers' reluctance to use technology in mathematics classes might be related to the types of problems they use. In general, those problems are exercises and often are not seen as a means to develop students' problem solving abilities. What kind of problems do teachers need to select in order to use technology efficiently? Current curriculum reforms recognize that the use of technology plays an important role in students' learning of mathematics. Technology is used as a generic term that includes the use of various types of software, hand calculators (symbolic and graphic), and other devices. In this paper, we reflect on basic principles that promote a new culture based on visual understanding and problem solving using technology. Current curriculum reforms recognize that the use of technology plays an important role in students' learning of mathematics. Technology is used as a generic term that includes the use of various types of software, hand calculators (symbolic and graphic), and other devices. Each type of technology might offer particular advantages for students to develop special features of mathematical thinking. Thus, there is interest in exploring ways in which the use of technology becomes relevant in dealing with mathematical tasks.

(Fri July 12, 11:00, HS 4)

Meira Hockman (School of Mathematics, University of Witwatersrand, Johannesburg, South Africa), 36mhh@cosmos.wits.ac.za

**Dynamic Geometry Systems as Teaching Tools in Teacher Education**

In South African schools, synthetic geometry with its strict order of theorems following definitions following axioms, dominates the curriculum. The senior scholars are initiated into the statement-required to prove-proof format for theorems and problems in the hope that this discipline will stand them in good stead for life skills or for abstract mathematics. In either case the aim is to generate logical and sequential thought. In contrast, the junior students are encouraged to explore ideas using low technological tools such as compass, straightedge and templates. The students explore concepts visually, prior to formulating definitions. The students subsequently take an active part in the formulation of definitions and theorems. Dynamic geometry systems allow a similar process of visual stimulation to precede formal axiomatic constructions at secondary and tertiary level, in particular in programmes for teacher education. Through these systems the students can explore complex ideas and theorems, investigate change and invariance, conjecture and hypothesize prior to or in the place of engaging with the formal rigorous mathematics. Introducing new concepts in this way and encouraging reflection may also produce the insight needed to understand the dichotomy between visualisation and sequential thought or "intuition" versus "rigour" when learning and teaching geometry.

(Fri July 12, 12:00, HS Ph)

David Jeffrey, Michel Beaudin, Rob Corless (U Western Ontario, Canada), djeffrey@uwo.ca

**Row reduction for computers**

We describe why computer users should ask for Turing factors, rather than reduced row echelon form or LU decomposition. We then demonstrate a program for the TI-89 and illustrate the applicability of Turing factors by solving some typical problems that have caused difficulties in our lectures.

(Fri July 12, 16:00, HS Ph)

Asha Jindal (National Council of Educational Research and Training, New Delhi, India), asha\_jindal@hotmail.com

**Using IT Tools For Interactive Exploratory Learning**

An application computer program can be used by a subject teacher for creating need based interactive learning environment in the classroom. Various concepts can be explained in graphical mode in addition to numerical mode. Variables can be changed just by click of mouse and in turn cause effect relationship can be investigated on computer monitor. The paper illustrates how a school teacher, having no knowledge of high level programming language can simulate situations related to curricular subject in a classroom with a spreadsheet program which will make learning richer, flexible, interactive, exploratory and hence more meaningful. The illustrations are exemplar and will serve the purpose of follow up activities/ exercises what a student has already learnt in traditional classroom.

*(Fri July 12, 16:30, HS 1)*

Sandy Scaffetta Johnson (University of Oklahoma, USA), johnson@ou.edu

**Using the Internet in a Graduate History of Mathematics Course**

This presentation will demonstrate the possibilities of using the Internet for supplemental research and dissemination of information gathered and displayed by graduate students enrolled in a History of Mathematics course. The presentation will include links, examples of research, examples of projects, methods of disseminating information, historical electronic timelines, and more. The research data is a combination of traditionally and electronically gathered information, formulated for display and access over the Internet.

*(Fri July 12, 13:30, HS 4)*

Peter Jones (School of Mathematical Sciences, Swinburne University, Hawthorn, Australia), pjones@swin.edu.au

**Exploring the potential of a hand-held computer algebra system using a classic problem**

In this presentation, the classic 'volume of a box' problem will be used to illustrate the alternative solution strategies available to students when they have everyday access to graphics calculators with symbolic algebra capabilities. In the process, and using the traditional pencil and paper solution as a benchmark, an attempt will be made to identify those mathematical skills that remain critically important when students have everyday access to such technology. This is of relevance to future curriculum development when graphics calculators with symbolic algebra capabilities become the norm in the classroom.

*(Wed July 10, 14:00, HS Ph)*

Stephan V. Joubert, Temple H Fay (Technikon Pretoria, South Africa), joubertsv@techpta.ac.za

**Dimensional Analysis and DERIVE**

We promote Dimension Analysis as a technique that promotes better understanding of the role of units and dimensions in mathematical modeling problems. Our student base consists of undergraduate students from the Science and Engineering Faculties who generally have one or two semesters of calculus and some linear algebra as part of their curriculum. Because of "In Service Training" which is an integral part of their education, they have a reasonable understanding of the link between theory and practice in their particular industry, but manipulating mathematical formulae is not necessarily a strong point. Dimensional Analysis involves both dimensionless products and linear algebra and, because of the latter, this branch of mathematical modelling was, until recently, beyond the reach of most of our undergraduates. However, we have found that we can blend the skills of a good technologist with the use of computer algebra systems to successfully teach Dimensional Analysis to these undergraduates.

*(Wed July 10, 14:30, HS 4)*

Marissa Justan (AMA Computer University, Quezon City, Philippines), mpjustan@amaes.edu.ph

**Programming Permutation Multiplication in Derive**

Various studies on interconnection networks have gained interests in recent years. The quality of an interconnection network is often investigated by assessing its degree and diameter. These properties are important as a network's diameter corresponds to the worst communication delay for broadcasting messages in the network while the degree describes the largest number of connections made to an individual processor. Recent developments of technologies have given us the prospect to new approaches in both the research and the teaching of the subject. One approach is computing the diameter by permutation multiplication. However, there are not many tools to support this process. In this paper, we shall illustrate permutation multiplication in the context of computing the diameter of an interconnection network and we shall show a Derive function that was developed to assist in teaching the concepts to the undergraduate level.

*(Wed July 10, 14:00, HS 4)*

Djordje Kadijevich (Mathematical Institute, Serbian Acad. of Sciences and Arts, Belgrade, Yugoslavia), djkadij@mi.sanu.ac.yu

**Towards a CAS promoting links between procedural and conceptual mathematical knowledge**

Promoting links between procedural and conceptual mathematical knowledge is an important goal of mathematics education that is by no means easy to achieve. As regards CAS-based mathematics education, such a state may partly be caused by some limitations of the available CAS environments. By examining a sample of expressions, functions, equations and inequalities, underlined are possible requirements for a CAS promoting links between the two knowledge types. The requirements suggest

how these environments may be refined in years to come to produce a CAS that is more pedagogically oriented.

(Fri July 12, 16:30, HS 4)

Gert Kadunz (Universität Klagenfurt, Austria), gert.kadunz@uni-klu.ac.at

**Möglichkeiten und Grenzen von Modulen bei DGS-Einsatz**

Die einschlägige fachdidaktische Literatur zum Einsatz von DGS im Mathematikunterricht untersucht besonders die Bedeutung und Verwendung des Zugmodus. Der Konstruktion und der differenzierten Verwendung von Modulen (Makros) wird weniger Augenmerk geschenkt. Der Vortrag stellt eine mögliche Klassifikation der Verwendung von Modulen vor, beschreibt das Verhältnis dieser Verwendung zur Verwendung des Zugmodus und weist auf Grenzen des Moduleinsatzes hin.

(Fri July 12, 11:00, Sem Ph)

Stefan Karnel (Institute für Software Technologie, TU Graz, Austria), skarnel@ist.tu-graz.ac.at

**Computer Algebra für Brüche -- angepasst an Ausbildungszwecke**

Rechnen mit Brüchen ist ein grundlegender Teil des Mathematikunterrichts. Erklärungen zum Bruchrechnen beziehen sich dabei hauptsächlich auf die bekannten Rechengesetze zum Kürzen, zum Addieren, usw. Computer Algebra Systeme (CAS) jedoch verwenden zum 'Bruchrechnen' nicht diese Gesetze, sondern Verallgemeinerungen des euklidischen Algorithmus. Im multivarianten Fall sind diese Algorithmen jenseits dessen, was im Mathematikunterricht normalerweise gelehrt werden kann.

Somit stehen die etablierten CAS-Algorithmen im Konflikt mit der Zielsetzung des ISAC-Projektes: Dieses zielt auf ein 'educational CAS', welches in Schritten arbeitet, die dem Benutzer auf Anfrage zur Erklärung mitgeteilt werden. Solche Erklärungen sollen beim Bruchrechnen also die bekannten Rechengesetze betreffen. Der Vortrag präsentiert 'simuliertes Rewriting' als Lösung des Problems beim Bruchrechnen, trotz Verwendung etablierter CAS-Algorithmen dennoch Erklärungen auf die bekannten, elementaren Rechengesetze zurückzuführen. Anforderungen seitens Interaktivität an 'simuliertes Rewriting' werden diskutiert, und die Implementierung dieser neu entwickelten Technik in ISAC demonstriert.

(Fri July 12, 16:00, Sem Ph)

Karl-Heinz Keunecke (Kiel, Germany), kh.keunecke@t-online.de

**Real Time Online Education for Teachers: Mathematics with Graphic Calculators and CAS**

Such an urgent need for educations of Mathematics teachers has never exist before, to introduce them in new technologies as graphical calculators, calculators with CAS, dynamical geometry software, CAS for PC's etc. It is on the other side more and more difficult to get exemption for an education for one or two days. In addition, less and less time to get used to so basically new aids and methods lets the increasing load through the school weekday. It is obvious, to complement conventional educations by internet based lectures. Since more as a year real time online lectures have been given at IPTS (among others responsibly for the teacher education in Schleswig-Holstein). In one- to two-hour lectures proven teaching units are introduced with the topics: Calculus, Linear Algebra and Theory of Probability.

It is less the matter of introducing the new equipment but it gets tempted to point out what problems can be solved and how the teaching changes if the new technology is used. At first participants register at [www.lernnetz-sh.de/13n/bildung1.html](http://www.lernnetz-sh.de/13n/bildung1.html) for the desired lecture and meet then in the "virtual classroom" at the arranged time. There is an acoustic connection between all participants and the speaker. On the screens of the participants computer turns up a PC emulation of the calculator which the instructor can operate by mouse click. The participants work on their own calculator and take contact with the instructor if they wish.

Parts from given lecture are shown and it is reported about the previous experiences

(Fri July 12, 15:30, HS 3)

Mihaly Klincsik, Csaba Sárvári (Pécsi Tudományegyetem, Pécs, Hungary), klincsik@witch.pmmf.hu

**How can we combine the CAS with authoring system tools to create a learning environment containing flexible feedback opportunities**

Using CAS as a powerful mathematical tool and the Toolbook Instructor object oriented authoring system we can create efficient learning environment. We are using two different types of learning-feedback modules:

- interactive feedback moduls are assisting in acquirement of the learning topics;
- exam modules.

The two different modules are forming a connected, coherent unit. The first form allows for the students to modify their answers and gives interactiv opportunity to get more information by communication with CAS worksheets. In order the teacher could be able to modify the learning process there are used log-file. This contains information about navigations and activities of the user. All the tests are available on the network. The second form is made for exam-environment, thus allows only one opportunity to give answer. In our presentation we perform the results about measuring made at our highschool-course, too.

(Fri July 12, 15:00, HS 1)

Heinz Klemenz (Gymnasium Wetzikon, Winterthur, Switzerland), hklemenz@geosoft.ch

**Plattformunabhängiges Computerwerkzeug für dynamische Raumgeometrie**

Die konstruktive Raumgeometrie ist in den letzten Jahren an den Gymnasien immer mehr in den Hintergrund gedrängt worden. Eine ersatzlose Streichung der konstruktiven Aspekte der räumlichen Geometrie wäre aber verhängnisvoll. Das computerge-

stützte Konstruieren mit Raumgeometrie-Software erlaubt dieser Tendenz entgegenzuwirken, indem durchaus anspruchsvolle Aufgaben mit entsprechenden 3D-Werkzeugen elegant gelöst werden können, ohne fundierte Kenntnisse der Darstellenden Geometrie zu besitzen. An vielen Gymnasien in der Schweiz hat sich diese Alternative mittlerweile auch in den Lehrplänen etabliert.

Im Vortrag wird das Programm GeometerPRO vorgestellt, welches die in gängigen 2D-Programmen (Euklid, Cabri, Cinderella, etc) enthaltenen Konzepte (Zugmodus, Ortslinien, etc.) in den Raum überträgt. Die Software ist in der Sprache Java programmiert, wird dadurch auf den schulrelevanten Betriebssystemen verfügbar sein und erlaubt den Datenaustausch zwischen verschiedenen Plattformen.

(Wed July 10, 14:30, Sem Ph)

Allen Klinger (UCLA, Los Angeles, USA), klinger@cs.ucla.edu

**Internet Exposition, Visualization, and Assessment**

Computer algebra systems relate to applied science in diverse ways. Yet mathematical structures reveal underlying truths, not the physical systems they model. Historic choices of notation constrain large populations to relative ignorance. This paper presents ways to overcome mathematical anxiety, indeed, instead stimulate active learning. The method is technology-based, depending on software (markup languages) and hardware (computer networks). The paper presents a many year set of experiences in preparing material and applying them in classes, individual directed studies, and for interaction with professionals (presentations, informal discussions). The conclusion concerns both quantitative and anecdotal evaluation of learning accomplished. The paper makes a case for the applicability of novel assessment strategies in technology-based delivery of mathematical knowledge (e.g., via distance learning forms of programmed instruction).

(Thu July 11, 11:30, HS 3)

Vlasta Kokol-Voljc (Pedagoska Fakulteta, Univ Maribor, Slovenia), vlasta.kokol@uni-mb.si

**Electronic Exercisers - A Way to the Pedagogical Calculator?**

Interactive tools such as calculators and computer software that are used for teaching mathematics are said to be developed more and more into "pedagogical tools." But what makes a tool a "pedagogical tool"?

One direction in which numerical calculators are developed in order to support the learning and teaching of mathematics in primary and middle schools are the "electronic exercisers". Are these machines pedagogical calculators?

(Thu July 12, 10:30, HS Ph)

Alan Krempler (Institut für Softwarertechnologie, TU Graz, Austria), alan@oeh.tu-graz.ac.at

**Zum Design eines elektronischen Arbeitsblattes für Mathematik**

Algebra Systeme bieten ausgezeichnete graphische Benutzeroberflächen, die Formeln darstellen wie von händischem Rechnen gewohnt, und die Rechenarbeit gut in Übersicht halten lassen. Unterlegt man der Rechenarbeit einen exakteren formal-logischen Rahmen, wie er im ISAC-Projekt vorgesehen ist, so entstehen auch an die Benutzeroberfläche zusätzliche Anforderungen. Der Vortrag diskutiert die folgenden Anforderungen an ein elektronisches Arbeitsblatt und demonstriert die in ISAC implementierten Lösungen:

- (1) Darstellung der logischen Zusammenhänge zwischen den Teilen einer Rechnung, z.B. die Weitergabe von Annahmen aus der Lösung eines Teilproblems, oder die Voraussetzung für die Anwendung einer Rechenregel (etwa: Nenner  $\neq 0$ ).
- (2) Integration von 'Browsern' über die Wissensbasis: Problemtypen zur Spezifikation einer Aufgabe, dafür verfügbare Lösungsmethoden, für gewisse Lösungsschritte anwendbare Rechenregeln sind zu sichten und auszuwählen.
- (3) Strukturierte Darstellung des spezifizierten Problems und der Lösungsschritte in dem Ausmaß an Details, die der Benutzer in einer bestimmten Situation braucht und einstellen kann.

(Wed July 10, 14:00 Sem Ph)

Jean-Baptiste Lagrange (IUFM de Reims France), lagrange@univ-rennes1.fr

**CASYOPEE, a symbolic environment for secondary students and teachers**

This presentation is based on the work of a French team of math educators, teachers and computer scientists supported by the National Institute for Pedagogical Research. It starts from a reflection about the integration of CAS (Computer Algebra Systems) in schools and the support that the development of new software environments might bring. We reflected on a general approach of the design of environments around a CAS Kernel and we are currently developing and experimenting two prototypes. Our approach is based on two principles.

The first principle is about the mathematical knowledge that the use of CAS should develop. CAS is basically an "expression transformer" and, to use CAS properly, students should think of functions from the recognition of equivalent expressions. In order that students get this "transformation sense", emphasis has to be put on algebraic proof.

The second principle deals with the learning situation. Our aim in a software development around a CAS kernel is to provide the student means to organise efficiently his(er) resolution of problems on functions and to give the teacher means to foresee student's resolution.

The two prototypes are oriented towards tasks. One is the study of functions, the other is the calculation of limits. Beside generic graphing, numeric and symbolic facilities, they include specific capabilities for each task and for algebraic proofs.

Extensive parametrisation functionalities are offered to help the teacher in the preparation of the student's resolution.  
(Fri July 12, 13:00, HS 1)

Josef Lechner (Amstetten, Austria), lejos@nol.at

**"Three vector products" or "Everything you always wanted to know about the equation  $x^2+1=0$  (but were afraid to ask)"**  
There are some (more or less isolated) topics in school maths, which are not very popular and therefore sometimes neglected. Different vector products, trigonometric addition formula, complex numbers and algebraic structures of numbers or mathematical objects respectively, belong to these topics.

There exist two vector products in school maths: dot product (with dimension 1) and cross product (with dimension 3). A product with dimension 2 is missing. Such an operation exists in form of multiplication of complex numbers, but this multiplication is rarely seen as a vector product or even compared with other vector products. This is a pity, because this would be a unique opportunity to help pupils to give meaning to complex numbers by CAS and DGS-visualisations.

By discussing the above mentioned equation the lecture is intended to show surprising and amazing connections between different (important) mathematical branches. You find this result if you expand your search for solutions of equations to  $\mathbb{R}^2$  and if the vector product of dimension 2 (let's call it "circle product") is used for common multiplications.

Finally a short summary of the connections between all the stated items is given.

(Thu July 11, 10:00, HS 1)

Carl Leinbach (Gettysburg College, PA, USA), leinbach@cs.gettysburg.edu

#### Growing Ideas with a CAS

We are living in a computing environment that stresses Object Oriented Programming. While the user of a particular object may have some idea about how it works, most of the details are hidden from the user. In Mathematics the idea of hiding information about a process is seen as a violation of the goals of mathematical pedagogy. However, as mathematics teachers, we do hide information from our students. We have been doing it for years! We do it in the form of presenting simplified models that take into account only that part of the real world process that can be modeled and solved using the mathematical theory and techniques the student is capable of applying. This form of information hiding may be more dangerous than that hidden by the computer scientist. It leaves the students with an incomplete understanding of the underlying process that they are attempting to describe. It subverts one of the reasons for studying mathematics - providing a better understanding of our world and how it operates.

In this presentation we will consider a case of the flight of a batted baseball (a ball of diameter approximately 7.6 cm made of thread wrapped around a cork core and wrapped in a stitched leather covering). We will begin the investigation assuming that the flight of the ball is a parabola. However, data collected on the flight of a batted ball find that the parabolic model does not describe this situation. A cubic curve gives a good approximation to particular curves, but the coefficients tell us little or nothing about the underlying phenomenon. To really understand the phenomenon we will need to construct a physical model that involves rates of change. This is where the information hiding comes in to play. Most students do not have the background to do this. The CAS the students have at their disposal can help them construct their model and give a general result that can predict the flights from arbitrary initial inputs. The net result is that students can do the modeling process, and the CAS can generate the results. Thus, we will watch the idea grow from a simple, but incorrect, quadratic curve model through the attempt to use regression techniques to gain understanding to the (still simplified) model using rates of change.

(Wed July 10, 13:00, HS 1)

Eberhard Lehmann (Rückert-Oberschule Berlin, Germany), mirza@snafu.de

#### Mathematikunterricht mit Parametern - schon in der Sekundarstufe 1

Computeralgebrasysteme eröffnen zahlreiche neue Möglichkeiten für einen neuartigen Mathematikunterricht. Dazu gehört auch die Fähigkeit von CAS, Module (Bausteine) mit Parametern zu verarbeiten, beispielsweise für magische Quadrate. Diese Module führen zu einigen neuartigen Arbeitsmöglichkeiten, die Parameter schon frühzeitig in das Blickfeld rücken. Aufgrund der zahlreichen Formeln im Unterricht schon der Sekundarstufe 1 eignet sich dieser Ansatz sogar als eine Leitlinie für den Unterricht beider Sekundarstufen. In dem Vortrag werden die Aussagen mit konkreten Unterrichtsbeispielen verdeutlicht. Dabei werden u. a. auch "Tipps und Tricks zum Forschen und Entdecken an Bausteinen" gegeben.

(Sat July 13, 10:00, Sem Ph)

Peter Lüke-Rosendahl (Bismarckschule Hannover, Germany), PeterLR@web.de

#### Berührungskreis dreier Kreise am Beispiel der Sichel des Archimedes

Möglichkeiten, Grenzen und Verbindungen im Gebrauch von DGS und CAS werden exemplarisch vorgeführt. Gesucht ist der Kreis, der die drei Kreise der Sichel des Archimedes (Shoemakers knife) gleichzeitig berührt. Durch "Trial and Error" in DGS wird die Basis für eine mögliche Untersuchungsrichtung aufgezeigt, die dann mit CAS realisiert wird.

(Wed July 10, 11:00, Sem Ph)

Elvira Malitte (Martin-Luther-Universität, FB Mathematik/Informatik, Halle, Germany), malitte@mathematik.uni-halle.de

**Mit dem Sinus auf der Spur des Mondes**

Aus der Tageszeitung können Daten für Mondaufgang und Monduntergang gesammelt werden. Dabei stellt sich die Frage: "Wie verändert sich die Mondscheindauer im Laufe der Zeit?"

Ausgehend von realem Datenmaterial wird eine Aufgabensequenz vorgestellt, die schrittweise zu einer angemessenen mathematischen Beschreibung hinleitet. Sowohl die Aufbereitung der vorliegenden Daten, die zunächst nur im Stunden-Minuten-Format vorhanden sind, als auch die mathematische Auseinandersetzung mit der funktionalen Beziehung "Tag -> Mondscheindauer" stellen grundlegende Aspekte der Aufgabensequenz dar. Die grafische Veranschaulichung führt zur Idee der Sinuskurven-Anpassung, diese Idee wird Schritt für Schritt durchdacht und realisiert. Der Grafikrechner erweist sich dabei als angemessenes und leistungsfähiges Werkzeug, mit dem die Aufgabe zu einem interessanten, vielgestaltigen Auftrag wird. Die Aufgabe ist sowohl in der Phase der Erarbeitung der Sinusfunktion geeignet als auch zum Festigen, Wiederholen oder selbstständigen Üben. Verzichtet man auf die Anpassung einer Sinuskurve an die Ausgangsdaten, d.h. stellt man den funktionalen Zusammenhang als solchen in den Mittelpunkt, so ist eine Bearbeitung bereits ab Klasse 7 denkbar.

(Fri July 12, 10:00, Sem Ph)

Giora Mann, Nurit Zehavi, Shafik Halifa (Levinsky College of Education, Beit Chanan, Israel), giorama@macam98.ac.il

**Vectors in CAS - A Portfolio for Teachers**

In the local high school curriculum a course on vectors was introduced, about 15 years ago, in order to integrate algebra and geometry in an advanced level course. The teachers often complained, that "when students learn about the direction vector they tend to loose the direction". The aim of the MathComp project at the Weizmann Institute is to broaden opportunities for learning and to promote greater mathematical understanding by using a Computer Algebra System (CAS). Within this framework, we have developed a portfolio for teachers, Vectors in CAS. We will demonstrate how teachers learn to customize the portfolio to their privileges, in terms of the mathematical content, the explicitness of the text, and the visual representation. The examples are taken from a unit dealing with the distance between lines in 3D space, through various representations. The CAS environment enables students and teachers to reflect on previously learned related topics, as well as to formalize and appreciate the concept of the distance between two geometric figures.

(Wed July 10, 13:30, HS 4)

Alan McRae (Washington and Lee University, Lexington, VA, USA), mcraea@wlu.edu

**A New Perspective on Geometry: A web-based series of independent study courses**

The purpose of these courses, a portion of which is near completion, is to make publicly available a comprehensive series of web sites for discovery-based, interactive courses on geometry. They will be offered at Washington and Lee University as part of our independent study program in modern geometry, allowing the student to choose those branches of geometry of greatest interest to her. The courses are developed with the help of student research scholars during the summer months. Ideas and definitions at each step in the learning process will be both motivated and illustrated, sometimes via interactive and dynamic geometric tools such as Cinderella. The theory will then proceed to develop through problems, reading assignments, and projects. Some of these problems will take the form of missing steps in interactive exercises written in Java. The students will then add their own contributions to the web course as appropriate (a beautiful new solution, an interesting find in an old journal, a surprising application, etc.). I will illustrate this program by looking at the first completed course, which is on finite geometries. Work will begin soon on a course for geometric probability.

(Wed July 10, 13:00, HS Ph)

Frantisek Mraz (Department of Mathematics, University of South Bohemia, Budejovice, Czech Republic), mraz@pf.jcu.cz

**Project Oriented Problems with Maple in Teachers Training**

The educational system in the Czech republic has a good level, which has been maybe even very good as regards fact knowledge of pupils. The system has been, however, criticized because of too much of memorizing and routine work. As the process of education has been usually inertial, a required improvement should be done, apart from other things, by some changes in teachers training. The presentation gives an information about project oriented problems which have been used in teachers training curriculum at our department. To improve our approach the experience of other participants would be useful. The idea is to give projects which solution needs to combine knowledge of different courses. The next goal is to stimulate experimentation and modelling, and to provide students with the space for their own discoveries. Therefore, several projects include randomly generated or parametric data. Students should construct problems with their own input data, too. To reach the above goals Maple has been used as it facilitates experimentation and enables to extend significantly the range of problems that cannot be solved successfully by hand calculation.

(Fri July 12, 12:00, HS 3)

Walther Neuper (Institut für Softwaretechnologie, TU Graz, Austria), neuper@ist.tugraz.at

**Re-engineering von Algebra-Systemen zum Mathematik-Lernen**

Derzeit verfügbare Computer Algebra Systeme (CAS) bringen die white-box Phase des Mathematikunterrichtes in Schwierigkeiten: Lehrer müssen den Schüler verbieten, das Studium von Konzepten durch CAS-Knöpfe Drücken abzuschneiden.

Das ISAC-Projekt zielt auf die Entwicklung eines Softwaresystems, das Mathematik als Methode des schrittweisen Lösens von Problemen aus Technik und Wissenschaft erfahrbar macht, wobei jeder Schritt begründbar ist. Diese Schritte betreffen die Spezifikations- wie die Rechen-Phase, und sie können sowohl automatisch vom System vorgeführt werden, als auch vom Studenten eingegeben (und vom System überprüft) werden. Diese Zielsetzung von ISAC erfordert ein Re-engineering von CAS, und auch gewisse Erweiterungen. Der Vortrag präsentiert die grundlegenden Entwurfsentscheidungen samt den Prinzipien ihrer Realisierung durch Methoden der symbolic computation, diskutiert die Vorteile und Beschränkungen aus diesen Entscheidungen, und demonstriert ausgewählte Beispiele mittels der bereits in ISAC implementierten Funktionalitäten.

(Wed July 10, 13:00, Sem Ph)

Walther Neuper, Andreas Griesmayer, Alan Krempler, Stefan Karnel (IST TU Graz, Austria), neuper@ist.tugraz.at  
**Re-engineering Algebra Systems for Education**

Currently available algebra systems (CAS) tend to sabotage the 'white-box' phase of teaching mathematics --- teachers must forbid students to shortcut studying the concepts by 'pushing a button'. The isac-project is dedicated to the development of educational software which supports a major part of learning math: modeling, specifying and solving problems in steps which the user can input interactively (checked by the system) and which can be suggested by the system interchangeably. These goals involve re-engineering functionalities of CAS. The talk presents isac's design decisions on the kind of steps and their realization by concepts of symbolic computation (matching and rewriting), discusses the advantages and limitations of the design decision, and demonstrates the features implemented by isac so far.

(Sat July 13, 12:00, HS 1)

Norma A Noguera, Mutindi Ndunda (California State Univ, USA), nnoguera@csulb.edu

**Making Mathematics Accessible for All Students Using Technology**

A common idea between minority and under-represented students who struggle with the learning of mathematical knowledge is that mathematics is a luxury that they cannot afford. Mathematics is a strategic keystone for the economic and cultural development of a nation. In addition, mathematics is a part of any new technological invention. When it is well taught it gives the child the opportunity to develop not only critical thinking skills, but also to learn about the connections between mathematics, science, social studies, etc. The presenters will share their research results working with CAS and Geometer's Sketch Pad at the middle, high school and college level.

(Sat July 13, 11:30, HS Ph)

Esther Openheim (Levinsky College, Kfar-Saba, Isreal), esther\_o@netvision.net.il

**Using CAS for Developing Students' Reflection and Reasoning - A Case Study on the Subject of Polynomial Interpolation**

Computer-Based environments challenge instructors to design new tasks for mathematics education, because it turns out to be unreasonable to continue to emphasis mathematics' instruction only on teaching skills (symbol manipulation), when a powerful supportive tool such as Derive5 (a mathematics assistant) can perform better and quicker. For the purpose of enhancing students' mathematical knowledge, and communications skills, and develop their advanced mathematical thinking we developed a sequence of CAS-based assignments for high school and pre-service students. Derive5 serves as a convenient dynamic tool for making complicated and tedious computation, and so release the user to devote himself to more essential aspects of the mathematics: raising conjectures, investigations, reflection and reasoning. The topic "Polynomial Interpolation" was chosen because of its' mathematical richness, and its' applicability to the current computer world. To rouse the student curiosity we start the workshop by presenting a stimulating question: 'Do you have any idea how a computer calculates the values of a given built-in function such as  $\sin(x)$  so quickly and so accurately?'

By observing and documenting students' laboratory work we followed their cognitive paths from visual interpretation of the approximation process to formal reasoning. Dealing with this issue of Polynomial Interpolation exposes the way polynomial interpolation is actually used for the purpose of building "simple" built-in computer functions. It added a new viewpoint on the basic idea of approximation and the values of adjacent solutions. At the end of the workshop students are able to construct acceptable approximations for a built-in function justifying any product they might have made. The challenging questions that are posed during the workshop stimulate discourses about various assumptions, which lead to further research work. Consequently it appears that the new CAS-based tasks open for learners a window on new intellectual connections and supports students in building on their own meaning in ways, which converge towards mathematical ways of knowing.

The paper discusses the underlying didactical principles of the workshops' design, convey the goals of two assignments, and describe the lab work of two high school students including their reflection and reasoning on their research process.

(Sat July 13, 12:00, HS Ph)

Tatyana Oleinik, S Gorkova, V Yevdokimov (Ukraine), olejnik@kgpu.sa.net.ua

**Development of Decision-Makers Competencies**

This paper presents the results of special courses given to undergraduate students with mathematics and computer science as subjects. A general purpose of our study theoretical framework is to mould teachers' competencies as reflective decision-makers and lifelong learners. It is evident that today the problem of changing a person's social level demands special attention: only a widely educated person with SOFT SKILLS is able to flexibly restructure the direction and essence of his or her activity.

According to this, students have to learn new learning techniques that will question how they currently decide problem and even what they currently believe about learning and decision-making. This is very important for posing questions, analyzing and at last understanding of what is happening. In other words, master specialists have to find clear and fruitful ways to characterize and carefully consider possibilities or alternatives for action and thoughtful assessment of choices made and implemented.

*(Fri July 12, 16:30, HS 3)*

Dusan Pagon (University of Maribor, Slovenia), dusan.pagon@uni-mb.si

**Analysis of simple branching trees with TI-92**

In the complex plane we start at the center of the coordinate system with a vertical segment of the length one unit. Then we turn to the left and to the right for a positive angle  $t$ , not exceeding 90 degrees, and add two new segments of length  $r < 1$ . The last two steps are repeated infinitely many times, to obtain a self similar fractal object, which we call simple branching tree (SBT). Typical questions about SBT are like: at what relation between  $r$  and  $t$  the branches of our tree will meet (overlap), and what will be the boundary curve, when there is no overlapping. The integration of algebraic and geometric tools that graphic calculator TI-92 possesses, allow us to find the answers to most of these questions. For instance, by summation of geometric progressions, an explicit connection between  $r$  and  $t$  is found for the boundary case and the obtained polynomial equations are solved numerically.

*(Wed July 10, 11:30, HS 3)*

Roger Peck (California State University, Bakersfield, USA), rpeck@csu.edu

**Teaching Statistics on the Internet**

This presentation will discuss how the Internet can be incorporated into a statistics course. The presenter has been using the Web in all his courses for the past eight years and has been teaching courses completely on the Web for the past four years. Using the Internet when teaching Statistics can have benefits to student learning. For example, since the interaction between students and faculty is almost entirely written, questions and answers are better thought out than when the spoken word is used. Student interaction can be facilitated by using email or discussion groups.

There are many data sets available on the Internet that can be used in the classroom for teaching or projects. This presentation will explore some well-known sites where data can be found and used in an introductory statistics course.

*(Fri July 12, 13:00, HS Ph)*

Michael Pemberton (The University of Queensland, Brisbane, Australia), mrp@maths.uq.edu.au

**CAS-aided Mathematical Modelling: an immersion course using PBL**

This paper reports on a course for biological and physical science students where Maple was used in all forms of teaching - in all lectures, tutorials and exams - something which has not been done before. All topics were introduced by real life problems which led both to discussions of the associated theory and computer implications. The course had five chapters -

- (1) Problems of optimization, where four problems introduced students to the need for understanding the difference between local and global maxima and minima, using Maple procedures to see the dependence of optimum solutions on parameters, approximations and the need for Maple for algebraic simplification and computation.
- (2) Biological and business models - epidemics, daily temperature, seasonal variation and trend, additive and multiplicative models; the Economic Order Quantity (EOQ) models, sensitivity analysis and discount models - using Maple to both set up and solve these problems.
- (3) Functions of several variables, using Maple for graphs and contours of functions of two variables, partial derivatives; relative and percentage error with many examples from physical chemistry and biology.
- (4) The Area Problem - how to approach calculating areas, integration techniques, numerical integration and extensive use of Maple to find harder integrals.
- (5) Differential and difference equations - models of exponential increase and decrease; growth by generation; migration and seasonal growth; Logistic growth; models of epidemics like AIDS; Future value, annuities and mortgages - using Maple's dsolve and rsolve to solve and then plot solutions for these problems. Students learned the necessary Maple as they went along and each tutorial was in the computer laboratory where they learned some new Maple and then used it in real life examples. In lectures Maple was used extensively and we, as a class, often discussed tutorial problems that had problems and how to overcome them. The exams were in front of the computer and students could bring along their Maple manual that I wrote especially for them and any other notes they might want.

Evidently, with this approach, it is possible to discuss a wide and often difficult range of problems with students who are not mainstream mathematicians very successfully and all students reported that they had at last understood the need for theory to underpin these ideas, whereas before it had just been a boring and irrelevant subject.

*(Sat July 13, 11:00, HS 1)*

Dennis Pence (Department of Mathematics, Western Michigan University, USA), dennis.pence@wmich.edu

**Good Things with Piecewise-Defined Functions on the TI-92Plus**

All too often, piecewise-defined functions are used to show discontinuity in a mathematics course, and then they never appear again. This leaves the mistaken impression that only "bad things" happen when you work with such functions. We will explore

some "good things" that can be done with piecewise-defined functions. This material will be appropriate for precalculus, calculus, and linear algebra courses.

There are three ways to specify a piecewise-defined function on the TI-92 family of CAS calculators. If there are only two or three pieces, the when( command generally provides a simple way to do this. Functions defined by a single when( command or two nested when( commands are displayed nicely in pretty print in the home screen. Expressions involving the when( command can be differentiated nicely (the pieces have the same defining logical tests), but integration does not work as well (because these defining logical tests are not necessarily well ordered). More than three pieces are most easily implemented in a function-type program using the IF-THEN-ELSE IF-ELSE IF.....ELSE control structure. Unfortunately we lose the nice display in pretty print, symbolic differentiation, and integration because there can be no symbolic understanding of what you might have done inside a program. A third way involves the sign( function, which is the derivative of the absolute value function. This built-in piecewise-defined function oddly cannot be symbolically differentiated but integrates nicely.

*(Wed July 10, 11:00, HS Ph)*

Werner Peschek (University of Klagenfurt, Dept. of Didactics of Mathematics, Austria), werner.peschek@uni-klu.ac.at

#### Modules, Black Boxes and the Principle of Outsourcing

Modules are important tools for human thinking, modularity is an important strategy for structuring and organizing individual as well as social knowledge. This strategy has also highly influenced the development and organization of computers and occurs here in different forms.

It is remarkable, that this fundamental idea was rather seldom taken up and analyzed in the didactical CAS-discussion up to now (contributions of H. Heugl, E. Lehmann or E. Schneider are rather exceptions). One reason for this reserve might be the feeling of uncertainty about the question, if it is acceptable (from a didactical point of view) to use CAS-modules without knowing their internal operational sequences ("black boxes"). In the so called "White-Box/Black-Box-Principle" (B. Buchberger) this is opposed, the "Principle of Outsourcing" (W. Peschek) comes to a positive answer.

Within this lecture, I will try to point out some aspects of modularity, which could be relevant for the use of computers in mathematics education (and lead to a sophisticated answer for the question mentioned above), I will present different occurrences and forms of modules and discuss E. Schneider's categorizing of CAS-modules and last but not least I will reflect on possibilities of a sensible use of CAS-modules in mathematics education.

*(Thu July 11, 11:00, HS Ph)*

Irina Peterburgsky (Dept of Math and Computer Science, Suffolk University, USA), irpeterbur@aol.com

#### Informal Learning and Game Elements in College Mathematics

Informal learning including storytelling and game elements can be alive and entertaining, engaging and challenging. It evokes students' curiosity and imagination, and builds up their intuition. Not only it is the best approach in teaching math to the small children but often a very successful way to open minds of adult learners with math anxiety and poor math background. This is the strategy that gives to many of our college students a rare chance to reconsider their skeptical attitude to mathematics, the chance to appreciate its elegance, power, and harmony. While being the most pleasant and enjoyable way for students to learn, it requires a lot of enthusiasm, creativity, and efforts on the part of their instructor.

In the present talk we are sharing a number of games, "math stories", and elements of informal learning which help our undergraduate students grasp a variety of mathematical ideas.

*(Fri July 12, 16:00, HS 3)*

Vladimíra Petrašková, Jindřich Klůfa (Univ of South Bohemia, Budějovice, Czech Republic), petrasek@pf.jcu.cz

#### Teaching mathematical analysis with Maple.

Presently there is a discussion in the Czech Republic concerning the role of computer in teaching of mathematics. Supporters of mathematical programs believe that students are unnecessarily overloaded with theory and that they try to reduce it. On the contrary, opponents of computer teaching refer to a mindless application of these programs, which results in the fact that the students are convinced that every thing can be algorithmized. There is such a reality that mathematical programs can enable a student to solve respective problem. However without knowledge of respective theory, the student is not always able to reach a correct solution. Many times, with the help of computer, he can also come to a meaningless conclusion. The content of the lecture is a model lesson of the workshop of mathematical analysis with the application of mathematical software, i.e. of Maple program.

*(Fri July 12, 15:00, HS 4)*

Valentyna Pikalova (Kharkiv State Pedagogical University, Ukraine), vpikalova@hotmail.com

#### The Role of Macros in Teaching Geometry: Developing Students' Algorithmic Skills

Dynamic Geometry Software (DGS) enables students to define such objects as points, lines (or segments, rays and vectors, circles or arcs) and construct geometric figures that depend on them by classical construction tools -- straight edge and compass, linear transformations etc. But one of the most important issues is that learner can create new geometric tools by using macro techniques. On one hand additional tools gave student new opportunities in problem solving. On the other hand macro constructions in DG environment involves learner into the framework of algorithms. Students are taught to decompose complex algorithms into units for constructing geometrical objects and record macros. Also analysis of capabilities of pre-

recorded macros by tracing greatly improves learners' activity. Development of macro constructions plays an important role in forming student's algorithmic culture as well as in efficiency of solving geometrical tasks. The article focuses on the methodological questions of macros implementation in teaching and learning geometry in secondary school.

*(Fri July 12, 11:30, HS 3)*

David Pountney (Liverpool John Moores University, UK), d.c.pountney@livjm.ac.uk

**Assessing Engineering & Science Mathematics in the presence of a CAS**

Since at least the early 1990s, academics have advocated that the presence of a CAS will dramatically affect the way we teach and assess mathematics for Engineering and Science students at University. However in many cases, the change has not been as dramatic as forecast and indeed a CAS often gets the blame for mathematical ills of students. This paper looks at reasons for this and reconsiders the future for CAS use for such students, based on a consideration of the mathematical skills (as perhaps opposed to subject knowledge) that we wish such students to acquire. Specific examples are used to illustrate the ideas.

It is conjectured that assessment issues remain a significant stumbling block to progress and that this pertains not only to Universities but to pre-University 16-19 year old mathematics education also. Again illustrative examples are described to show how an approach based on assessment of mathematical skills rather than techniques could lead to changes in teaching and learning where the use of a CAS is a natural consequence for students.

*(Sat July 13, 10:00, HS 1)*

Antonio R. Quesada (The University of Akron, Ohio, USA), aquesada@uakron.edu

**On Reforming an Introductory Linear Algebra Course With a TI-92**

This talk will present the results of a three-semester experience with reforming a traditional Linear Algebra course via the use of a TI-92 symbolic calculator. The principles that guided the experience were:

1. Going from the concrete to the abstract,
2. A hands-on, inquiry based approach was used, and
3. The use of technology to: a) facilitate exploration of many relevant applications, b) to discover through experimentation important theoretical results, c) to bridge over cumbersome calculations thus gaining access to relevant algorithms traditionally excluded from our Linear Algebra curriculum, and d) to reduce the time spent and numerical mistakes while students are learning the implementation of algorithms.

The basic themes of an introductory Linear Algebra course were maintained, but the approach was changed to a matrix-oriented course. The emphases were on conceptual understanding, on problem solving, and on providing a variety of motivating applications. These were aimed to demonstrate the pervasive use of Linear Algebra in many applied disciplines and its role as an essential tool for scientists in industries. Moreover, the use of special purpose programs and scripts allowed for the inclusion of new topics.

*(Wed July 10, 14:00, HS 1)*

Eugenio Roanes-Lozano, Eugenio Roanes-Macías, Matilde Villar-Mena (Univ. Complutense de Madrid, Spain), eroanes@fi.upm.es

**Linking The Geometer's Sketchpad 3 with DERIVE 5**

Both Computer Algebra Systems (CASs) and Dynamic Geometry Systems (DGSs) have reached a high level of development. Powerful Euclidean Geometry packages exist in some CASs, but CASs have incorporated neither mouse drawing capabilities nor dynamic capabilities. Meanwhile, the well-known DGSs do not provide algebraic facilities. After analysing theoretically the situation and after approaching the problem from Maple, we have now developed a new DERIVE Euclidean Geometry package (denoted ParamGeo) plus an external translator (both freely available from the authors). They make it possible to draw a geometric configuration with the mouse (using The Geometer's Sketchpad 3) and to obtain the equations of the drawn configuration in DERIVE-acceptable syntax. To obtain complicated formulae or to perform automatic theorem proving from sketches are direct applications.

*(Fri July 12, 11:00, HS 1)*

Csaba Sárvári (Pécs, Hungary), sarvari@witch.pmmf.hu

**CAS-basierte Lehrplan - Lehrplan-basierte Modularisierung mit CAS**

Anwendung CAS verändert, ergänzt den Inhalt der Lehrpläne und auch die benutzten Methoden. In dieser Hinsicht können wir sagen: CAS beeinflusst die Lehrplangestaltung. Grundsätzlich ist aber die andere Richtung: alle unsere Verfahren sollen vom Lehrplan, von den Lernzielen determiniert, reguliert werden. Modularisierung ist ein wichtiges und effizientes Strukturierungs- und Organisationsprinzip des kognitiven Wissens. Prozess der Modularisierung mit CAS ist ein dynamisches Verfahren, in dem Mass der Auslagerung soll immer von Zielen des Lehrplanes bestimmt werden. Im Vortrag wird Themenkreis der Modularisierung aus dieser Hinsicht behandelt und mit Beispielen aus der Hochschulunterricht demonstriert werden.

*(Sat July 13, 12:00, Sem Ph)*

Julie Saunders, Bill Blyth (Royal Melbourne Institute of Technology, Australia), julie.saunders@rmit.edu.au

**Using CAS as a Pedagogical Tool in the Training of Pre-service Mathematics Teachers**

The Mathematics department at RMIT delivers a mathematics course to pre-service secondary mathematics teachers. The course aims to strengthen the students' understanding of the more advanced secondary Mathematics curriculum topics and to

extend their knowledge in areas such as complex variables, differential equations and power series. Use of the computer algebra system Maple is incorporated into the delivery of the course. The objective here is to use the CAS as a tool to help consolidate the students' understanding while also exposing them to the potential of such tools for their own teaching in the future. Here, we present examples from the Maple worksheets used and summarise the findings of a feedback survey consisting of questionnaires, student journals and structured interviews. We found that the students favoured the use of pen and paper exercises alongside Maple work on the Complex Variables topic. In a Maple worksheet demonstrating Newton's Law of Cooling, we found that the CAS was able to provide the scaffolding in the form of computation of derivatives, allowing the students to focus on the modelling process. The students also highlighted the visualisation aspects of a worksheet on Taylor Series as revealing to them more clearly the concept of Radius of Convergence.

(Sat July 13, 10:30, HS 4)

Wolfgang Schlichthorn (Georg-von-Langen-Schule, Holzminden, Germany), wolschl@t-online.de

#### Anforderungen an ein schüler- und damit auch kundenorientierte CAS

Meine Schüler (bei denen ich DERIVE einsetze) besuchen die Sekundarstufe I (Berufsfachschule Wirtschaft) sowie die Kaufm. Berufsschule (Industriekaufleute). Ich werde folgende Probleme, die sich nicht nur auf DERIVE beziehen, ansprechen:

1. Verständnishindernisse durch fremdsprachige Anweisungen, unnötige Amerikanismen und Sprachsprünge bzw. Sprachdoppelungen. Pädagogisch gestaltete Benutzerführung.
2. Die Nicht-Einhaltung der gesetzlich vorgeschriebenen bzw. kaufmännisch üblichen Datendarstellung stört den Unterrichtsablauf: statt 44,342,345.67€ muß es 44.342.345,67€ lauten. Außerdem wird die Datenübernahme (die automatisierte ist teilweise unmöglich) aus Anwendungen nur durch deren erneute Eingabe möglich, d.h. die Schüler kämpfen mit der Software und nicht mit dem Problem.
3. Einige mit DERIVE erfolgte Anwendungen aus dem Unterrichtsbereich

(Wed July 10, 12:30, Sem Ph)

Franz Schlöglhofer (Universität Linz, Austria), f.schloeglhofer@utanet.at

#### Wahrscheinlichkeitssimulationen zum "Ziehen ohne Zurücklegen"

Simulationen von stochastischen Prozessen sind in der Wahrscheinlichkeitsrechnung sehr wichtig. Man erhält zwar als Ergebnisse nur relative Häufigkeiten statt Wahrscheinlichkeiten und Mittelwerte anstelle von Erwartungswerten, kommt aber mit den Modellen oft weiter als es mit theoretischen Berechnungen möglich ist, weil nicht in jedem Fall schwierige kombinatorische Überlegungen angestellt werden müssen. Im Vortrag werden hauptsächlich Beispiele zum "Ziehen ohne Zurücklegen" behandelt. Dafür wurden Prozeduren in DERIVE 5 geschrieben, die als Grundlage für die Berechnungen im Unterricht direkt verwendet werden können.

(Wed July 10, 10:00, Sem Ph)

Karsten Schmidt, Anke Köhler (FH Schmalkalden, Germany), kschmidt@fh-sm.de

#### Einsatz von CAS und symbolischen Taschenrechnern an Hessischen Schulen

Es werden Ergebnisse einer im März 2002 durchgeführten Befragung an allen Hessischen Gymnasien, Real- und Gesamtschulen vorgestellt. In dieser Untersuchung geht es einerseits um die Erfassung des Ist-Zustandes bezüglich der PC-Ausstattung der Schulen und dem tatsächlichen Einsatz leistungsfähiger Taschenrechner im Mathematik-Unterricht. Andererseits soll festgestellt werden, wie gut sich Lehrkräfte mit symbolischen Taschenrechnern und CAS auskennen und wie häufig sie solche Hilfsmittel persönlich (z.B. zur Unterrichtsvorbereitung) benutzen. Darüber hinaus wird untersucht, ob die Lehrkräfte beim Einsatz von symbolischen Taschenrechnern und CAS mehr Vorteile oder mehr Nachteile sehen und ab welcher Klassenstufe sie leistungsfähige Taschenrechner im Mathematik-Unterricht einsetzen würden, wenn sie die Entscheidung selbst treffen könnten. Die Ergebnisse werden verglichen mit denen einer ähnlichen Befragung an allen Gymnasien, Regel- und Gesamtschulen in Thüringen, die im Frühjahr 2001 durchgeführt worden war.

(Wed July 10, 12:00, Sem Ph)

Reinhardt Schmidt (Christian-Weise-Gymnasium, Zittau, Germany), r.schmidt@sz-online.de

#### Einsatzmöglichkeiten des CAS Mathcad im Mathematikunterricht am Gymnasium

Erfahrungen aus einem Schulversuch "Computerunterstützter Mathematikunterricht" kurz: *CuMaU*. Lösung von "Standard-Aufgaben" im MaU mit verschiedenen Werkzeugen (GTR ohne CAS, TI-92, Derive, MAPLE, Mathematica, Mathcad), Praktizierung verschiedener didaktischer Prinzipien beim Einsatz des CAS Mathcad anhand von Beispielen. Zur Zeit liegen vielfältige Erfahrungen beim Einsatz von Mathcad in der Sekundarstufe I (Klassenstufe 9 und 10) vor.

(Fri July 12, 16:00, HS 4)

Edith Schneider (University of Klagenfurt, Department of Didactics of Mathematics, Austria), edith.schneider@uni-klu.ac.at

#### CAS and Experimental Learning

The availability of technological tools as CAS offers the possibility of outsourcing operative (algorithmical, routine) activities (which require often a very great deal of time and work) to the "mathematical expert" CAS. Thereby it gets possible to use experimental working also in maths classes in a sensful and adequate manner. Many concrete proposals for teaching mathema-

tics base on such experimentation (including keynotes as: investigating, exploring, discovering, heuristical approaches, ...). The black-box/white-box-principle formulated by H. Heugl refers to this possibility offered by CAS.

Within the lecture it is planned to scrutinize the significance of experimental working closely from the point of view of mathematical education of students of general educating high schools. From this perspective a few possibilities of experimental learning in CAS-supported maths classrooms (as for instance investigating characteristics of mathematical concepts, discovering mathematical rules, exploring CAS as a mathematical learning environment, ...) and their relevance for maths classes should be discussed exemplarily by means of concrete examples.

*(Thu July 11, 12:00, HS Ph)*

Peter Schofield (Trinity & All Saints College, Leeds, UK), p\_schofield@tasc.ac.uk

#### **Folding, Cutting and Joining DERIVE 5 Style**

This lecture describes how DERIVE 5 can be used to establish direct links between transformation geometry activities at Key Stages 1 and 2 of the UK National Curriculum for Mathematics (approx 5-11 yrs) and related topics in a Mathematics Subject Studies Module for Primary Education Degree Students.

The first form of KS 1\2 activity involves folding and cutting sheets and strips of paper to form attractive symmetry group (cut-out) patterns. Using a DERIVE 5 Users File 2D-&3D-Transformations.dfw (containing general-purpose tools for transformation geometry) these activities will be simulated in DERIVE's 2D-Plot Window. The second form of KS 1\2 activity involves constructing regular polyhedrons by joining together networks of their faces. Using 2D-&3D-Transformations.dfw the networks, transformations and polyhedrons will be constructed in a similar manner in DERIVE's 3D-plot Window. In particular, the cube and dodecahedron will be highlighted. The presenter will also explain how these can form challenging activities for the above degree course students.

*(Fri July 12, 14:00, HS Ph)*

Steven Schonefeld (Tri-State University, Angola, Indiana, USA), sschonefeld@mchsi.com

#### **Picturing a Complex Function (25 minute talk)**

In this paper, we explore some two-dimensional curves, which enhance our understanding of a complex function,  $f(z)$ . We follow the usual convention of letting  $z = x + iy$  (where  $x$  and  $y$  are reals and  $i$  is the square root of negative one). With this  $z = x + iy$  notation, we get a one-to-one correspondence between each point,  $(x, y)$ , in the Euclidean plane and each complex number. When  $z = x + iy$ ,  $x$  is called the real part of  $z$  and  $y$  is called the imaginary part of  $z$ . The DERIVE function  $RE(z)$  returns the real part of  $z$  and  $IM(z)$  returns the imaginary part of  $z$ . That is,  $x = RE(x + iy)$  and  $y = IM(x + iy)$ . When we plot the implicit curves given by  $RE(f(x + iy)) = 0$  and  $IM(f(x + iy)) = 0$ , the points of intersection will be the zeros of  $f(z)$ . We explore the families of curves given by  $ABS(f(x + iy)) = n$ , for  $n$  positive and  $PHASE(f(x + iy)) = k$ , for  $k$  between  $-\pi$  and  $\pi$ . Finally, we show the surprising result that these two families of curves are orthogonal.

*(Wed July 10, 11:00, HS 3)*

Max-Günter Schröfel (Gymnasium Antonianum, Salzkotten, Germany), Max-Guenter.Schroefel@fernuni-hagen.de

#### **Splines mit dem TI92 (DERIVE)**

Es werden zunächst zwei Anwendungsbeispiele für Splines vorgestellt:

- Glatte Flächen über Stützpunkte werden in der Autoindustrie verwendet. Dieses Vorgehen wurde im Unterricht nachempfunden werden, indem wir bei einer Motorhaube drei Stützpunkte und die Neigung der Windschutzscheibe festhielten. Daraus wurde die glatte Projektionskurve der Motorhaube mit Hilfe von Splines ermittelt.
- Zwei Autobahnstücke sollen mit einer Straße verbunden werden. Die Abfahrtspunkte, ein Umgehungspunkt und die Autobahnrichtungen werden festgehalten, daraus wird dann der Straßenverlauf bestimmt.

In der Theorie der Splines ist das Hauptziel das Gleichungssystem zu reduzieren, um es besser berechnen zu können. Dabei konzentriert man sich auf eine Unbekannte des Polynoms, die ändern werden dann mit Hilfe dieser berechnet. In den Beispielen wird bei der Berechnung der Splines das Lösen des umfangreichen Gleichungssystems dem TI 92 überlassen, die Arbeit für den Anwender besteht in der Erstellung der großen Matrizen. Im zweiten Teil soll nun gezeigt werden, wie die Spline-matrizen vom Rechner erstellt werden können. Es wird der systematische Aufbau der Spline-matrizen untersucht und kleinere Teilmatrizen aufgespürt, die letztlich auf drei  $4 \times 1$ -Matrizen und Nullmatrizen zurückgeführt werden. Die Spline-matrizen werden mit einer Hand voll Matrizenoperationen zusammen gesetzt. Nach der Lösung des Gleichungssystems werden dann die Splinepolynome bestimmt. Es wird ein Beispiel mit drei Punkte, der Anfangs und Endsteigung gegeben. Durch Programmierung könnte alles automatisiert werden.

*(Fri July 12, 14:00, Sem Ph)*

James Schultz (Ohio University, Athens, OH, USA), schultz@ohio.edu

#### **To CAS or not to CAS? -- that is NOT the Question!**

Building on a historical perspective of hand-held technology use, the presentation will provide examples of appropriate uses of CAS, as well as very similar examples where CAS is not the most effective tool. Finally it reexamines the initial question in light of these examples, leading instead to the questions "When to CAS?" and "With whom to CAS?". Examples will span mathematics for ages 12 to 19.

*(Fri July 12, 13:30, HS Ph)*

David Sjostrand (Elof Lindälvs gymnasium, Kungsbacka, Sweden), david@ydsa.se

**A Computer Integrated Mathematics Teaching at the Swedish Natural Science Program**

Powerful computers and easy to use mathematics soft wares enables a modern, fun and interesting mathematics teaching. Since autumn 2000 we integrate the use of computers in all math courses at the Natural Science Program at Elof Lindälvs gymnasium, Kungsbacka, Sweden. Students are working more with projects in a self dependent way. Traditional written tests have to some extent been replaced by project tasks. The students participate in compulsory national tests. We will present samples of project tasks. The use of computers in mathematics teaching has inspired us to have another view of mathematics teaching at secondary school level. We will argue that

- Students can attain a lot of understanding of graphs by using interactive Excel sheets created with the help of DERIVE.
- 3D-plotting can and should be included.
- Computer algebra makes it possible let students solve problems that are more demanding algebraically than earlier - they get the chance to see so much algebra that their understanding of it might increase.
- Once the students have understood the basic idea of derivatives you can let them solve differential equations numerically. Evaluation of definite integrals can be regarded as solving differential equations.
- Computer algebra and spreadsheets make analytical geometry more interesting and powerful - you can easily deal with complicated algebraic formulas and see that they really are true.
- The computational power of computers makes it possible to include mathematics that traditionally has not belonged to school mathematics in secondary school mathematics. Students can for example evaluate not only simple integrals but even double and triple integrals exactly and numerically.

*(Fri July 12, 15:00, HS Ph)*

Bunpot Suwannaprasert (Naresuan University, Thailand), bunpots@nu.ac.th

**Web-Base Instruction on Mathematics**

This study investigated the feasibility of utilizing existing technology as a mean for developing a prototype Thai Web-Base Instruction (TWBI) for teaching mathematics on the Internet. Results of this study indicated that it is possible to develop a TWBI by using every program concerned to produce the web sites for teaching mathematics. The results could also be applied to different instructional subjects.

*(Wed July 10, 10:30, HS 4)*

Eno Tonisson (University of Tartu, Estonia), eno@ut.ee

**Equivalence of equations and computer algebra systems**

In connection with the spread of computer algebra systems (and algebraic calculators), the natural question arises: how to change the requirements and emphases of mathematics syllabuses? One possible domain that might be given more consideration in the future is checking the equivalence of equations.

In this paper, we examine the solving strategies of school algebra equations and give much attention to equivalence checking. In order to find the solution of an equation, it is often necessary to change it to an equivalent equation. Sometimes it is recommended to use operations which do not guarantee equivalent equations (the extraneous solutions may appear, for instance). To what extent are the computer algebra systems applicable in checking the equivalence of equations? An overview is provided of the capabilities of the currently widespread systems (Derive, Maple, Mathematica and MuPAD). We also examine some possibilities of integrating checking the equivalence of equations in computer algebra systems more fully into the educational process. We present different schemes that describe the teacher's and student's activities in different situations.

*(Fri July 12, 10:30, HS Ph)*

Marlene Torres-Skoumal (Vienna International School, Austria), marlenes@aon.at

**Curricula and Assessment Model for the integration of CAS into the high school program**

The Vienna International School has been integrating computer algebra systems into its MYP (Middle Years Program) curricula for the past four years. Curricula and assessment criteria has been rewritten to reflect the changes in learning objectives brought about by this new technology.

This presentation will highlight the VIS model for the use of technology in classroom teaching and assessing. The model classifies curricula and assessment into three main categories: technology free, technology supported, and technology focused. The technology free parts attempt to assess the numerical and algebraic components of the curricula that are not obfuscated by computation. In addition, technology neutral topics are assessed under this caption.

The technology supported areas focus upon those numerical, graphical, or experimental skills which can be executed using analytical methods, but would be tedious and time consuming otherwise. Students can freely choose between paper and pencil methods taking time element into account, or can rely upon the technology to assist when necessary. Their judgment as to the appropriate use of the technology would form part of the assessment criteria.

The technology focused areas bring into the classroom possible concepts or problem types that are unapproachable or have no (accessible) analytical solutions. This would include sophisticated problem solving, investigations, and mathematical modelling.

Samples of assessment of the above three areas, including student samples, will be shared with the participants.  
(Wed July 10, 10:00, HS 1)

Lisa Townsley (Benedictine University, Lisle, USA), ltownsley@ben.edu

**Multimedia Classes: Can they ever be too much techno?**

Abstract: Have you ever had a 4-technology day in the classroom? Did your students witness you using the calculator, CAS, handouts and projection devices, with a side trip to the WWW? The speaker talks about some experiences she and her colleagues have had with multiple technology platforms in the classroom, including the good, the bad, and the ugly. The best stories are those of students presenting talks using multiple platforms.

(Wed July 10, 10:30, HS 3)

Hildegard Urban-Woldron (Pädagogische Akademie der Erzdiözese Wien, Austria), hildegard.urban@chello.at

**CBR und CBL im fächerübergreifenden Unterricht /Mathematik - Physik- Informatik**

Anhand einer exemplarischen Einführung wird der Umgang beim Messen mit einzelnen Sensoren und die anschließende Auswertung der Daten in Verbindung mit einem TI-Rechner erläutert. Experimentell ermittelte Daten werden nach ihrer grafischen Veranschaulichung der Theorie und dem Modell gegenübergestellt. Die Einsatzmöglichkeiten des CBR und CBL im Unterricht zum Experimentieren, Veranschaulichen, Analysieren, Modellieren und Interpretieren bereichern durch anwendungsorientierte Fragestellungen und fächerübergreifende Betrachtungen die Unterrichtsarbeit in hohem Maße. Beziehungen der Daten können numerisch und grafisch erforscht, mit mathematischen Funktionen assoziiert und mit repräsentativen Funktionen modelliert werden. Dabei kann die physikalische Bedeutung der einzelnen Parameter erkundet und in der Folge ein tiefergehendes Verständnis erzielt werden.

(Thu July 11, 11:00, Sem Ph)

Nelson Urrego (Universidad Javeriana, Bogota, Colombia), nurrego@javeriana.edu.co

**Recursive procedures and recursive functions using DERIVE**

In the formation of mathematicians and computer engineers, the study of recursive functions and recursive predicates constitute a very important subject because it allows to students to understand the reaches and limits of the mechanical procedures in a rigorous and abstract way. In order to make more effective this study and to motivate the students, the design and implementation of algorithms for solving problems related with Recursion Theory are proposed as pedagogical tools. For this purpose, such algorithms are represented with the help of DERIVE. This Mathematical Assistant allows to program recursive algorithms for the representation of interesting functions and procedures such as the generation of prime numbers and the implementation of 1-1 Primitive Recursive correspondence between  $N_n$  and  $N$ , among others. Some examples of these algorithms and the advantages of the use of DERIVE for its implementation are presented in this lecture.

(Wed July 10, 12:00, HS Ph)

Shin Watanabe (Tokai University, Japan), watanabe@scc.u-tokai.ac.jp

**Problem solving with graphic calculator**

In my mathematical lessons, I want to teach mathematics with problem solving style. I make the laboratory of mathematics in the school. In this room we use the mathematical tool that is graphic calculator. The method of the laboratory system is called the making an experiment in mathematics. The student makes the theorems that were made by the old mathematician. The mathematical experiment with graphic calculator is very important for students. They learn the subject of mathematics with enjoyment. In this lesson we study mathematics with discovery method. The student has the power of creativity. We show the making theorem in the classroom. We use the graphic calculator; so making theorems are algebraic style. And we show the tool is very important and what is mathematical tool?

(Fri July 12, 14:30, HS 4)

Shin Watanabe (Tokai University, Japan), watanabe@scc.u-tokai.ac.jp

**The mathematical laboratory with TI-89 and CBL**

We want to make the room of mathematical laboratory with TI-89 and CBL in every school. In mathematics lessons teaching mathematical knowledge is given to student. So we make the system of teaching which is making mathematics by student. For example, we connect the two subjects, math and physics using TI-89 and CBL. We teach mathematics with joy at mathematical lesson.

(Wed July 10, 15:30, HS 3)

Jon Sims Williams (Dept Engineering Maths, Bristol University, UK), Jon.Sims.williams@bris.ac.uk

**Consolidating Learning through Self-Help Testing**

In the popular culture right across Europe, mathematics is often seen as a hard subject that is largely irrelevant to the bulk of today's youth. When these same people get into employment however, they find they need more mathematics but now they must teach themselves or follow a distance-learning course.

Reading examples in textbooks may leave the reader feeling he understands, but it is only through being tested on unseen problems that a student can consolidate his understanding.

This paper describe a way in which such students can be offered the opportunity to test themselves. The TAL database of classified questions allows students to generate tests on the topics they are studying though a web-interface from home. These automatically generated tests allow students to validate their skills against a standard, eg the SEFI syllabus, and to get feedback on questions they cannot do. Often the hints provided in feedback on questions are not adequate, so we can direct the student using the question classification system to support material that will give them targeted help with background understanding that may be the cause of their difficulties.

*(Wed July 10, 11:00, HS 4)*

Otto Wurnig (University of Graz, Austria), otto.wurnig@uni-graz.at

**Using PC and TI-92 in teaching linear regression and correlation on different levels in the grammar school. (In German)**

After the recent reforms of the curricula in mathematics, statistics and the use of the computer were fixed in different grades of the grammar schools. This offers a good possibility to develop such difficult concepts as linear regression and correlation on different levels of precision.

In grade 8 the pupils make the first steps towards Two-Variable-Statistics. Without a computer the pupils of grade 8 plot the data  $x$  and  $y$  as coordinate pairs per hand. By doing so they sometimes realize e.g. that  $y$  has the tendency to be directly proportional to  $x$ . This makes the pupils try to draw a straight line which fits the points. If a computer is available in the classroom, you can demonstrate the solution of this problem easily with the help of a software program like MATHEASS. After the input of the coordinate pairs you get the function term of the approximation curve, the coefficient of determination, the correlation coefficient and the standard deviation together with the diagram.

It is very important not to take sets of data out of the school books only. I allow the students of grade 8 to work with their own data (length and mass) or help them to find an actual event (referendum "Veto against Temelin") to test the correlation. If all the pupils use the TI-92 the med-med regression is a new possibility to find the regression line: In this case it is only necessary to know about the concept of linear function. Later on, in grade 11 or 12, the students can find real data with the help of CBL (calculator-based laboratory) in an experimental way in Physics and find the regression type in Mathematics. With the computer or TI-92 the students can plot the two regression lines very quickly and with the two slopes they find a new aspect of the concept of determination.

*(Fri July 12, 13:00, Sem Ph)*

Yuzita Yaacob, Noraini Hassan, Khairina Atika Mohd Zawawi, Hasni Amiruddin (National Univ of Malaysia), yy@ftsm.ukm.my

**Interactive Learning - Mathematica Enhance Calculus (ILMEC) courseware: A Pedagogical Tool to Enhancing Calculus Education in Malaysia**

Traditional mathematics courses emphasize the learning of mathematics through rote work, memorization and mastery of hand methods of solving problems. Although this can result in creating a good human calculator, it is not conducive to in-depth and substantive understanding of mathematical concepts. In the last decade, there has been a revolution in the teaching and learning of mathematics, both within mathematics and in its related disciplines, due to the introduction of Computer Algebra Systems (CASs) such as Maple and Mathematica. However, an educational problem remains in smoothly integrating a CAS into the whole experience of teaching and learning of mathematics.

This project presents ILMEC, a multimedia courseware using a CAS, to serve three roles in the teaching and learning of Calculus. Specifically, the courseware can aid in the (i) mathematical concept and skill development by empower students to deal with multiple representations (numeric, symbolic and graphic), visualization, interactivity, experimentation and exploration, (ii) mathematical problem solving by enhancing the ability to focus on the process of problem solving instead of the computational aspect only and solve realistic problems instead of being restricted to contrived problems having "nice solutions", and (iii) mathematical reasoning by empower students to input different values in order to form conjectures and apply inductive reasoning, and motivate students to think logically to perform a desired task.

*(Fri July 12, 13:00, HS 4)*

Piotr Zarzycki (Department of Mathematics, University of Gdańsk, Poland), matpz@univ.gda.pl

**From Visualizing to Proving**

Benefits of visualizing mathematics by technology like TI 92+ and mathematically oriented software (DERIVE 5 and CABRI II) are undisputable. On the basis of some examples we would like to show that visualizing techniques could help students to analyze certain mathematical problems better and give them a strong support in finding formal proofs for considered problems. We present some tasks taken from school algebra, geometry, number theory and probability; for these problems it is easier for students to pass the bridge between visual and formal solutions. We conclude that the role of technology is essential in building up associations between graphs, drawings and other tools used for the formal proofs.

*(Fri July 12, 10:00, HS 1)*

## Workshops

Rüdeger Baumann (Gymnasium Ernestinum Celle, Germany), baumann-celle@t-online.de

### Algorithmische Geometrie mit Derive

An Beispielen aus der Analytischen Geometrie bzw. Vektorgeometrie wird gezeigt, wie sich dieses Gebiet der Oberstufenmathematik mit Hilfe eines Computeralgebra-Systems (etwa Derive) zu einer Algorithmischen Geometrie weiterentwickeln lässt. Die Schüler erstellen Konstruktionen in Gestalt von Algorithmen (bzw. Derive-Programmen) und beweisen deren Korrektheit. Es ist zu diskutieren, ob das (die Rechenmacht des) Computeralgebra-System(s) auch durchschnittlichen Schülern das selbständige Finden von Konstruktionen und Beweisen erleichtert oder allererst ermöglicht, wie diese sich von den herkömmlichen Beweisen und Konstruktionen unterscheiden und was vom "Geist der Geometrie" dabei noch übrig bleibt.

Die Teilnehmer sollen angeregt bzw. angeleitet werden, anhand von Aufgaben aus der Analytischen Geometrie / Vektorgeometrie bzw. Linearen Algebra eine (im Sinne des Vortrags) CAS-unterstützte Methodik des Konstruierens und Beweisens zu entwickeln.

(Thu July 11, 10:00, PC-Lab A)

Josef Böhm (DUG, Würmla, Austria), nojo.boehm@pgv.at

### Programming in DERIVE - Some Introductory Examples

In this workshop we will use some examples to inform the participants about the possibilities how to connect the power of a CAS with the flexibility of a programming language. Special impact is given how to treat local and global variables. The chosen examples will cover problems from within secondary school level, because it is our opinion, that programming should become - again - part of math education. We can do this now without changing the platform using DERIVE 5.

(Fri July 12, 14:00, PC-Lab A)

David Bowers (Suffolk College, Ipswich, UK), david.bowers@suffolk.ac.uk

### Computer algebra within a spreadsheet-style environment

Spreadsheets have long been a useful tool for analysing mathematical problems numerically, and computer algebra systems allow us to approach problems symbolically. In this workshop we demonstrate how the Texas Instruments TI-92 goes some way to incorporate computer algebra into a simple spreadsheet environment. Examples of some unexpected applications will be investigated.

Firstly the basic functions and operations of the TI-92 Data/Matrix Editor are reviewed. Then various activities covering a range of mathematical areas are proposed. The first of these deal with numerical applications of the type that may be familiar to users of spreadsheets in the mathematics classroom, and serve to illustrate the spreadsheet-style use of the Data/Matrix Editor. The subsequent activities introduce some features of computer algebra systems that hitherto have not been available within a spreadsheet environment.

Participants are encouraged to review the activities and their usefulness for demonstrating and investigating mathematical concepts in new ways, to consider the advantages and technical limitations of the Data/Matrix Editor for this purpose, and to speculate on the future development of algebraic spreadsheets.

(Wed July 10, 14:00, HS 3)

Bernnard Cunningham (Mott Community College, Vassar, Michigan, USA), bcunning@gfn.org

### Use the Program Editor of the TI-89/92 Calculator to write a program to solve cubic equations

The workshop will begin with a brief history lesson on the mathematicians that were responsible for the development of the cubic formula. To equalize all participants, programming the quadratic formula will be the first activity. This will address Input/Output (I/O) and Control (CTL) submenus as well as the capabilities of a word processor in copying and pasting. The cubic formula will then be analyzed and a plan of attack will be developed for the writing of this program. (The old flow chart plan of attack.) The participants will be allowed some time to write their programs. To close the activity, programmers will be allowed to show their programs. Now, if time permits, this workshop will turn to a lecture on the proof of the cubic formula.

(Fri July 12, 14:00, HS 3)

Homero Flores (Colegio de Ciencias y Humanidades-UNAM, Mexico City, Mexico), ahfs@servidor.unam.mx

### Geometric Proof in Upper Middle School

Regardless its complexity, it is relevant to teach Mathematical proof in Upper Middle Level (UML, grades 10-12), because it can foster the development of a critical and reflexive reasoning in our students. In this workshop I present a teaching proposal of geometric proof in UML based on Brousseau's Didactical Situations theory, with the aid of TI-92 calculators and Sketchpad.

The goal of the workshop is to reflect on the use of a-didactical situations in a CAS environment and the development of a reflexive thinking in the student, while attendants involve themselves in hands-on proof activities. Proposed duration: One session of two hours, or two sessions of 90 minutes, depending on the TI-92 and Sketchpad skills of the attendants.

(Fri July 12, 12:30, HS 3)

Wilfried Herget (Univ. Halle-Wittenberg, Halle, Germany), herget@mathematik.uni-halle.de

**A Picture Tells a Story of Well Over 1,000 Words**

"Maths means calculating!" - All right, but that is certainly not the whole story: there is far more to mathematics than that!

In this talk, some unusual open-ended problems are presented which may be used at secondary school. Here, calculating is not the main focus of attention, but rather the steps before all the calculations: "Here is a situation. Think about it!" (Henry Pollak)

The true value of such a problem and its solution lies in the pleasure you have derived from courageously taking your own steps, from being creative and bold in search of the right answers, and from experiencing what it is like to find a rough solution by yourself, instead of reverently looking up to the answer, or getting somebody else to work it out for you.

(Fri July 12, 10:00, HS 3)

Carl Leinbach (Gettysburg College, PA, USA), leinbach@cs.gettysburg.edu

**Programming with TI-Interactive!**

TI-Interactive is a general purpose tool for doing mathematical investigations. It allows students to use a PC-based platform to perform many of the functions of a TI graphing calculator: do scientific calculations, define variables and evaluate formulas, do statistical analyses, graph functions, do parametric and polar plots, etc. In addition, it allows them to connect to any website containing data and copy that data into a TI-Interactive! work sheet for statistical analysis. It also allows for connectivity with a TI-calculator to copy data collected by the TI-Ranger or TI-CBL. In short, it is a powerful tool with much of the functionality of a TI-83 for use in a mathematics classroom. TI-Interactive! also has a powerful Computer Algebra System that contains most of the functionality of the TI-89/92 plus. This all is packaged in a way that allows students to create nicely formatted and displayed interactive documents that can display their results. What is not as well known is that TI-Interactive! also allows a user to write programs in a large subset of TI-Basic.

In this workshop participants will be given an introduction to programming within the context of TI-Interactive! . After a brief introduction to the available programming structures and some of the programming operators that are available and others that are not available, they will write some simple functions starting with an interval characteristic function (which can be done without programming). The workshop will continue to develop some more interesting functions and also show how TI-Interactive! can graph these user-programmed functions (a process that is not immediately obvious). The culmination of the workshop will be an illustration of how to draw the graph of a slope field and an Euler and Runge-Kutta approximation to a function defined in terms of a rate of change.

(Fri July 12, 12:00, PC-Lab A)

Palmira Mariz (Vienna International School, Austria), pmariz@vis.ac.at

**Dynamic Geometry in Classroom**

Geometer's Sketchpad is a powerful Dynamic Geometry Software that became an important tool for the study of high school geometry. This software enable teachers to use different approaches to geometry topics and provides a wide range of teaching strategies: explorations of open-ended problems, guided investigations, construction activities,.... GSP is also a useful tool to produce teaching materials such as worksheets. The aim of this workshop is to introduce participants to the main features of GSP through exploration of sample classroom activities. Previous knowledge of the program is not required.

(Wed July 10, 12:00, Praktikum A)

Al Maturo (Leysin American School, Switzerland), amatur@las.ch

**TI-83 Plus Calculator Workshop**

TI-83 Plus Calculator Workshop. This workshop is designed to teach one how to use the Texas Instruments TI-83 & TI-83 Plus Calculators with hands on exercises. Al Maturo of the Leysin American School will demonstrate the capability of the TI calculators as well as offer exercises to allow the user, you, a chance to learn 'hands on' how to use this classroom tool. Al Maturo is a teacher of IB and AP Mathematics and uses the TI calculators when appropriate in the classroom. The workshop is designed for the new user, or for those with little experience with the TI Calculators. Although we will use the TI 83 Plus, this workshop is compatible with TI-82 and TI-83 Calculators. Calculators will be provided for those without one, but feel free to bring your own.

(Fri July 12, 10:00, HS 6)

Al Maturo (Leysin American School, Switzerland), amatur@las.ch

**TI-92 Plus Calculator Workshop for Beginners**

This workshop is designed to teach one how to use the Texas Instruments TI-92 Plus Calculator with hands on exercises. We will also use the TI-89 Calculators, but there is no difference in their capability; they only look different. Al Maturo will demonstrate the capability of the TI calculators as well as offer exercises to allow the user, you, a chance to learn 'hands on,' how to use this classroom tool. He uses the TI calculators when appropriate in the classroom. The workshop is designed for the new user, or for those with little experience with the TI-92 Plus. Calculators will be provided for those without one, but feel free to bring your own.

(Wed July 10, 12:30, HS 3)

Al Maturo (Leysin American School, Switzerland), amatur@las.ch

**Statistics with List Editor with the TI-89/TI-92Plus Calculator (Workshop for Intermediate & Advanced Users)**

This workshop highlights the ability of the TI-89/TI-92Plus to perform Statistical Functions using the Flash Program: Statistics with List Editor. Statistics with List Editor is a free Flash program which offers more computing capability while enhancing user-friendliness. The workshop is designed for intermediate & experienced users, but new users that attend the TI-92 Workshop for Beginners will have enough knowledge to follow along. Al Maturo will offer you exercises and a chance to learn "hands on" how to use this classroom tool. Calculators will be provided for those without one, but feel free to bring your own. Current users of the TI-89/TI-92Plus and TI-83Plus are encouraged to attend.

*(Thu July 11, 10:00, HS 3)*

Gay L. Nixon (Lynnwood, Washington, USA), gaynixon@juno.com

**Absolute Value: Geometrically, Algebraically and Technologically**

We will work for useful and enduring understanding of absolute value by:

- 1) geometrically by analyzing the equations on number lines; (We will make sense of the symbolism of absolute value by use of words and graphs.)
- 2) algebraically by solving the equations in a logical manner; (We will solve the absolute value equations algebraically using some easily remembered formulas.)
- 3) technologically by verifying results on the TI-83+ calculator. (We will graph the equations on the calculator, sketch them on the worksheet in provided grids and analyze them.)

*(Fri July 12, 12:00, HS 6)*

Regis Ockerman (Steken, Belgium), regis.ockerman@pi.be

**Drawing Julia Fractals on the TI 92**

Considering the function  $f(z) = z^2 + c$ , in the complex plane, is an interesting application of the theory of complex numbers. It is also the source of the Julia Fractals. After a small theoretical introduction, we start with the example  $c = -0.75$  which brings us to the San Marco fractal. Taking advantage of the easy way of working with complex numbers and the graphic possibilities on the TI 92 we develop a program for drawing the fractal. This works pretty fast, considering that a TI 92 is not a Pentium III or IV. This program can draw Julia fractals for any (complex) value of  $c$ , belonging to the "apple basket of Mandelbrot". Some familiarity with the TI 92 is an advantage. Programming knowledge is not required.

*(Sat July 13, 10:00, HS 3)*

John Olive (The University of Georgia, Athens, GA, USA), jolive@coe.uga.edu

**Exploring the Dynamic Geometry of Calculus with Geometer's Sketchpad 4.0**

Version 4 of the Geometer's Sketchpad Dynamic Geometry software (GSP 4) has a new powerful algebraic interface whereby functions can be entered as algebraic expressions and graphed on coordinate axes. These expressions can be constructed from dynamic measures or parameters, associated with geometric constructions or just free-standing. The symbolic derivative of any function can also be obtained and graphed in GSP 4. However, unlike other graphing software, the function graphs can be used as geometric objects. Thus tangent lines to a free point on the curve can be constructed as a true tangent or as the limit of a secant line. Such a construction can provide students with insights concerning the derivative of a function as the slope of the tangent line at any point on the function. Using the new iterate transformation, polygons can be constructed to illustrate the area under a curve and provide calculations of the Riemann sum integral.

The workshop will introduce participants to these powerful new features of GSP 4 and involve them in exploring these concepts of calculus through custom-made dynamic sketches.

*(Fri July 12, 16:00, PC-Lab A)*

Philip Oostenbroek (Marist College Ashgrove, Australia), philoostenbroek@yahoo.com

**Creating Favourable Attitudes in Mathematics in 12 year old boys**

A series of activities and projects designed to create favourable attitudes in primary mathematics through the use of basic facts and geometry. Students will appreciate that Mathematics can be fun and is used in all aspects of life.

Basic computer programs using Draw, Excel and Word and Powerpoint can be used.

*(Wed July 10, 10:00, PC-Lab A)*

Michael Pemberton (The University of Queensland, Brisbane, Australia), mrp@maths.uq.edu.au

**Modelling Air-resistance using Maple**

This workshop takes the participants through an assignment given to my engineering and science students to investigate how to model air-resistance. First students drop plastic bottles containing various quantities of water from buildings of various heights to test which of four models is most applicable - air-resistance is proportional to (1) the velocity (2) the square of the velocity and whether (3) per unit mass or (4) directly. Participants will be given the data collected by my students and use Maple to solve each differential equation and test the solutions for validity.

For the second part, students use the same bottle, hang it on an elastic string, pull it down and release it. Students then observe the subsequent motion and try to fit the best model to it - linear or non-linear. Participants will use Maple to solve each differential equation, animate them and then watch to see how well the animation matches the physical motion.

*(Wed July 10, 12:00, PC-Lab A)*

Alfred Rosing (Ahlen, Germany), AHRosing@t-online.de

#### Modellierung von kontinuierlich ablaufenden Prozessen durch diskret dynamische Beschreibungen

Es soll demonstriert/untersucht werden, wie in der Umwelt ablaufende Prozesse, die gewöhnlich durch e-/ln-Funktionen beschrieben werden, durch iterative Modelle (diskrete Dynamik) beschrieben werden können, die den realen Vorgang gut nachbilden. Als Beispiel verweise ich auf meinen Beitrag, der Grundlage für den Workshop sein soll, in den TI-Nachrichten, Ausgabe 02/2001. Das Beispiel zeigt wie verschiedene Bereiche der Mathematik - z.B. Geometrie, Algebra, Algorithmik - mit einander in Beziehung treten.

*(Sat July 13, 11:30, HS 3)*

Peter Schofield (Trinity & All Saints College, Leeds, UK), p\_schofield@tasc.ac.uk

#### Some General-Purpose Tools for 2D- and 3D-Linear Transformation Geometry with DERIVE 5

The concept of a General-Purpose Tool for plotting in DERIVE 5 is that of an instruction that will generate expressions for plotting in both the 2D-Plot Window and 3D-Plot Window (depending upon the information in its argument places). In addition, the tool can be applied to (almost) all of the types of objects that can be plotted. It can also process either a single object or a list of such objects. The tools are contained in a DERIVE Users File

2D-&3D-Transformations.dfw.

The workshop activities will be based around the following general-purpose tools: PARA - plots objects with respect to user-selected intervals (when required); TRAN - linearly transforms objects using a 2x2 or 3x3 matrix; DIS - displaces objects using a row or column displacement vector; INV - inverts objects with respect to the 2D- or 3D-axes; STR - stretches objects in the directions of the 2D- or 3D-axes; ROT - rotates objects about the origin (2D) or about an axis through the origin (3D); REF - reflects objects in a line through the origin (2D) or a plane containing the origin (3D).

The file 2D-&3D-Transformations.dfw also contains instructions for drawing a selection of 2D- and 3D- laminas and objects, including: unit squares, triangles, regular polygons and circles; three point triangles and parallelograms; stretched polygons and ellipses; unit cubes, tetrahedrons, spheres, cones and double cones; four point tetrahedrons and parallelepipeds; cuboids, spheroids, etc.

*(Fri July 12, 10:00, PC-Lab A)*

Alla Stolyarevska (Kharkov, Ukraine), stolyare@altavista.com

#### The Using of Prolog Programming Language as a Computer Algebra System

Last years the tendency of genesis of Prolog programming language is observed. Prolog as widely known Lisp allows to manipulate a formula symbolically. We used the Prolog while processing knowledge and the data submitted as symbolical structures.

Within a framework of the course of informatics at Kharkov G. S. Skovoroda Pedagogical University the following topics are considered: realization of recurrent and iterative algorithms (calculation of sums, fast degree, Fibonacci numbers), operation with polynomials, other tasks. The course is oriented on those students of mathematics and physics faculty who are getting acquaintance with the methods of declarative programming.

The introduction of methods of declarative programming was carried out in different academic groups within several years for the students of different age categories. And the greatest successes at training the undergraduates were achieved.

*(Wed July 10, 10:00, Praktikum A)*

Johann Wiesenbauer (Technische Universität Wien, Austria), J.Wiesenbauer@tuwien.ac.at

#### Primality Testing and Factoring Large Numbers with DERIVE

According to Gauss "the problem of distinguishing prime numbers from composite numbers and of resolving the latter into their prime factors is known to be one of the most important and useful in arithmetic." Today these problems have also become crucial with respect to some highly topical encryption methods, such as e.g. the RSA-cryptosystem. This could also serve as the starting point to deal with the following basic questions in the classroom:

- (1) How can we recognize whether a given integer  $N$  is prime or not?
- (2) How can we find a nontrivial factor of  $N$  in the latter case?

As for the first problem, many computer algebra systems e.g. DERIVE make a compromise by using probabilistic primality tests. For example, the so-called Miller-Rabin test (essentially based on Fermat's Little Theorem) is used in most CAS. Other important probabilistic primality tests use certain properties of Lucas sequences. Of course, if absolute security is needed, one has to resort to deterministic primality tests, which are far more timeconsuming in general though. In particular, deterministic primality tests for Mersenne and Fermat numbers are given.

The factorization problem is much tougher by comparison. Here are some of the factorization methods which I am going to talk about in my workshop:

- (1) Trivial division: Before applying more complicated methods one should search for small prime factors of the given number  $N$ .

- (2) Pollard's "rho" method: One of the simplest factorization methods which also most CAS make use of. It is based on the famous „birthday paradox“.
- (3) Pollard's (p-1)-method: Very efficient, if p-1 factors into many small primes for some prime factor p of N.
- (4) The Elliptic Curve Method (ECM): This method was announced by Lenstra, Jr., in 1985, and uses facts from the theory of Elliptic Curves.

*(Wed July 10, 14:00, PC-Lab A)*

## Posters

Temple Fay, Lawrence Mead (Technikon Pretoria, South Africa), thfay@hotmail.com

### A Radiative Reaction Model

We investigate a nonrelativistic model for the classical electrodynamic equations of motion for a radiating particle. This model is suitable for discussion in a beginning differential equations course and beginning physics course on particle dynamics. It is of interest as it is linear of third order and the third derivative acts as a damping term. Boundedness of the solution becomes of critical importance and we determine conditions for bounded solutions in the unforced and simple sinusoidal forcing cases. We also discuss the sensitivity of numerical solutions. Our discussion suggests further investigations suitable for undergraduate research projects or small group investigations.

Darja Silan, Selma Stular, Julijana Palcic (Ljubljana, Slovenia), Darja.Silan@guest.arnes.si, julijana.palcic@guest.arnes.si

### The Dynamic of Some Biological Activities with the TI-CBL System

The teachers of natural sciences at the Joze Plecnik Grammar School Ljubljana-Slovenia, are participating in the international project Comenius. We would like to contribute our experiences using CAS environments in particular TI-92. We believe that mathematical experiments supported by the TI-CBL system can be enhanced by various aspects of the biological and physical problems.

Our poster represent a carbon dioxide production during germination of various seeds, metabolism of yeast (*Saccharomyces* sp.), larval and adult stages of mealworm (*Tenebrio molitor*).

In biological laboratory exercises for students, these experiments are basic but also very demonstrative for better understanding how the different organisms do their metabolism of energy rich substances during aerobic respiration.

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### Conference Office Opening Hours (@ Strudlhofgasse 4):

Tuesday 9<sup>th</sup> of July: 16:00-19:00 / 4pm-7pm  
 Wednesday 10<sup>th</sup> of July: 7:30-16:00 / 7:30am-4pm  
 Thursday 11<sup>th</sup> of July: 8:00-14:00 / 8am-2pm  
 Friday 12<sup>th</sup> of July: 8:00-17:00 / 8am-5pm  
 Saturday 13<sup>th</sup> of July: 8:00-13:00 / 8am-1pm

